D. SKORDEV, *An axiomatic treatment of recursiveness for some kinds of multi-valued functions.* Let $\mathscr{F}$ be a semigroup with an identity I. The elements of $\mathscr{F}$ and their multiplication will play the roles of functions and composition. Let $\mathscr{C} \subseteq \mathscr{F}$ (the elements of $\mathscr{C}$ will be "the single-valued constants"). As variables ranging over $\mathscr{F}$ and $\mathscr{C}$ we shall use $\varphi, \psi, \theta, \chi$ and x, y respectively. Let $\varphi, \psi \mapsto (\varphi, \psi)$ and $\chi, \varphi, \psi \mapsto (\chi \supset \varphi, \psi)$ be a binary and a ternary operation on $\mathscr{F}$ ("pairing of the function values" and "definition by cases") and assume that $\forall x \forall y((x, y) \in \mathscr{C})$. Let $L, R \in \mathscr{F}, T, F \in \mathscr{C}, T \neq F$ and let the following equations hold identically: $xy = x$, $(\varphi, \psi)x = (\varphi x, \psi x)$, $(\varphi x, I)\theta = (\varphi x, \theta)$, $(I, \psi(x))\theta = (\theta, \psi x)$, $L(x, y) = x$, $R(x, y) = y$, $(\chi \supset \varphi, \psi)x = (\chi x \supset \varphi x, \psi x)$, $(I \supset \varphi x, \psi x)\theta = (\theta \supset \varphi x, \psi x)$, $\theta(\chi \supset \varphi, \psi) = (\chi \supset \theta\varphi, \theta\psi)$, $(T \supset \varphi, \psi) = \varphi$, $(F \supset \varphi, \psi) = \psi$. Let $\leq$ ("the inclusion relation") be a partial order on $\mathscr{F}$ such that: (i) the three

given operations on $\mathscr{F}$ are monotonic with respect to $\leq$, (ii) $\forall\varphi\forall\psi(\forall x(\varphi x \leq \psi x) \rightarrow \varphi \leq \psi)$ and (iii) each chain $\mathfrak{M}$ in $\mathscr{F}$ has an upper bound ψ satisfying the condition

$$\forall\varphi\forall x\forall\chi(\forall\theta(\theta \in \mathfrak{M} \rightarrow \varphi\theta x \leq \chi) \rightarrow \varphi\psi x \leq \chi).$$

The iteration of φ controlled by χ is by definition the least solution θ of the equation $\theta = (\chi \supset I, \theta\varphi)$. A "function" ψ is called recursive in some "functions" $\psi_1, \cdots, \psi_n$ iff ψ can be obtained from $I, L, R, T, F, \psi_1, \cdots, \psi_n$ by means of the three given operations on $\mathscr{F}$ and iteration. For this notion of recursiveness we prove a normal form theorem, an enumeration theorem and the first and second recursion theorems.

Consider a set M together with a pairing mechanism J, L, R on it and let $c_i \in M_i \subseteq M$ $(i = 0, 1)$, $M_0 \cap M_1 = \varnothing$ (using the notations from [1], we can for example take M to be the set B^* corresponding to an arbitrary set B and take $J = \lambda st.(s, t)$, $L = \pi$, $R = \delta$, $M_0 = B^0$, $M_1 = B^* - B^0$, $c_0 = 0$, $c_1 = 1$). We obtain a model for the given system of axioms taking $\mathscr{F}$ to be the set of all partial multiple-valued mappings of M into M with the natural rule of composition and the natural partial ordering and taking $\mathscr{C} = \{\lambda s.c : c \in M\}$, $(\varphi, \psi) = \lambda s.J(\varphi(s), \psi(s))$, $(\chi \supset \varphi, \psi) = \lambda s.\{t: (\chi(s) \cap M_0 \neq \varnothing \wedge t \in \varphi(s)) \vee (\chi(s) \cap M_1 \neq \varnothing \wedge t \in \psi(s))\}$; $T = \lambda s.c_0$, $F = \lambda s.c_1$; in the special case which corresponds to [1] our notion of recursiveness will be equivalent to absolute prime computability. Other models can be obtained by taking $\mathscr{F}$ to be the set of the partial mappings of M into M, or by considering fuzzy or probabilistic mappings of M into M. We can also take $\mathscr{F}$ to be the set of all pairs $\langle D, f\rangle$, where f is a partial multiple-valued mapping of M into M and $D \subseteq \operatorname{Dom} f$, and define multiplication by $\langle D_0, f_0\rangle \cdot \langle D_1, f_1\rangle = \langle D, f_0 f_1\rangle$, where $D = \{t: t \in D_1 \wedge f_1(t) \subseteq D_0\}$. Then for a suitable interpretation of the rest of the primitive notions we again obtain a model for the considered system.

REFERENCE

[1] Y. N. MOSCHOVAKIS, *Abstract first order computability.* I, ***Transactions of the American Mathematical Society,*** vol. 138 (1969), pp. 427–464.