D. SKORDEV, *On the detection of periodic loops in computational processes.*
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In [3], the following notion of cyclic loop and an efficient method for detection of such loops were considered (as seen from [1, §3.1]; another practically usable method has been proposed by R. Floyd). Let P be a function whose domain is the set $\mathbf{N}$ of natural numbers, and, for all $t, \bar{t} \in \mathbf{N}$, the implication $P(t) = P(\bar{t}) \Rightarrow P(t+1) = P(\bar{t}+1)$ holds. Let $t_1 \in \mathbf{N}$. Then it is said that *a cyclic loop is present in P at the moment* t_1 iff $P(t_1) = P(t_0)$ for some $t_0 < t_1$.

If P is as above, then $\{(t, \bar{t}) \in \mathbf{N}^2 : P(t) = P(\bar{t})\}$ is a congruence relation in the algebra $\mathcal{N} = (\mathbf{N}, ')$, where $'$ is the successor function. Of course, the above definition of cyclic loop and the detection methods can be reformulated in terms of congruence relations in $\mathcal{N}$. As the next paragraph shows, this turns out to be convenient for a certain new application to the deterministic case of recursive algorithms in the sense of [2].

Let E and M be sets (the *set of labels* and the *set of memory states*), and let some $\alpha \in E$ and $a \in M$ be chosen (the *initial label* and the *initial memory state*). An *E-instruction over M* is, by definition, a triple (λ, f, w), where $\lambda \in E$, f is a partial mapping of M into M, and $w \in E^*$ (i.e. w is a string of labels). Suppose a set R of E-instructions over M is given such that $\operatorname{dom} f_1 \cap \operatorname{dom} f_2 = \varnothing$ whenever (λ, f_1, w_1) and (λ, f_2, w_2) are two distinct elements of R. Let S be the least defined partial mapping of $E^* \times M$ into itself such that $S(v\lambda, x) = (vw, f(x))$, whenever $v \in E^*$, $(\lambda, f, w) \in R$, and $x \in \operatorname{dom} f$. Define $t_1 r t_0$ (*at the moment* t_1, *a repetition of the situation from the moment* t_0 *is present*) to mean that $t_0, t_1 \in \mathbf{N}$, $t_0 < t_1$, and there are $u, v, w \in E^*$ and $x \in M$ satisfying

$$S^{t_0}(\alpha, a) = (vu, x), \qquad S^{t_1 - t_0}(u, x) = (wu, x).$$

Then $r \cup r^{-1} \cup I_{\mathbf{N}}$ is a congruence relation in $\mathcal{N}$.

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