Randomized First Order Computability
(abstract)

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In his path-breaking paper [1] Yiannis Moschovakis introduced and studied two important notions of first order computability for partial multiple-valued functions on an arbitrary set, namely the notions of prime and search computability. The unprejudiced way of operating with multiple-valued functions in that paper stimulated the present author to study a generalization of some part of computability theory to other function-like objects. The generalization was announced in the paper [2], and a detailed presentation was given in the book [3] (as well as in an earlier book in Russian). Both computability notions studied in [1] turned out to be instances of the considered general one, hence the results of the above-mentioned generalization of computability theory apply to them.

A lot of other examples were indicated, and some of them are of a probabilistic nature. One such probabilistic example was examined more thoroughly, namely a case of randomized computability in the set $\mathbb{N}$ of the natural numbers (cf. Theorem 1 on pp. 251–252 of [3]). The function-like objects considered in that particular case are non-negative real-valued functions θ such that $\mathrm{dom}(\theta) = \mathbb{N}^2$ and for any x in $\mathbb{N}$ the inequality

$$\sum_{y=0}^{\infty} \theta(x, y) \ \leq \ 1$$

holds. Any such θ is regarded as a representation of a randomized partial function in $\mathbb{N}$ such that for any x and y in $\mathbb{N}$ the number $\theta(x, y)$ is the probability of returning the value y when x is given as argument, and the above sum is the probability of returning some value at all for the given x. The random number generator used in the computations is supposed to produce 0's and 1's with equal probabilities, and a characterization of the computable objects θ is given in terms of recursive enumerability. Intuitively, these objects represent the randomized partial functions in $\mathbb{N}$ that are computable by means of serial computational procedures allowed to use the random number generator in question.

Considerations of a similar kind (assuming the use of the same random number generator) will be presented in the talk, but about the case of non-negative real-valued functions θ such that $\mathrm{dom}(\theta) = (B^*)^2$ and for any x in B^* the inequality

$$\sum_{y \in B^*} \theta(x,y) \ \leq \ 1$$

holds, where B is an arbitrary set, and B^* is constructed as in [1]. The objects θ that are computable in some given partial functions in B^* will be studied. Intuitively, these objects represent the randomized partial functions in B^* which are computable by means of serial computational procedures allowed to use the given partial functions and the considered random number generator. No straightforward analog of the result concerning randomized computability in $\mathbb{N}$ holds for them without some additional assumption. However, they will be characterized through prime computability of the function value on the base of the value of the argument and certain finite sequences of 0's and 1's, the probability of returning any concrete function value depending on the set of all such finite sequences which lead to that value for the given value of the argument. Of course, since the computability investigated in this example is a particular instance of the general notion from [2, 3], the corresponding generalized results of computability theory are applicable also to that kind of computability.

References

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