Scott-Continuous Embeddings and Learning Families of Algebraic Structures from Text

Alexandra Soskova, Sofia University

We study Scott-continuous embeddings between classes of structures closed under isomorphisms based on generalized enumeration operators as an analogue of the Cantor-continuous embedding. Here we take into consideration only the positive atomic diagram $\mathcal{D}_+(\mathcal{A})$ of a structure $\mathcal{A}$, and not the basic (positive and negative) atomic diagram $\mathcal{D}(\mathcal{A})$. We prove an effective version of the Lopez-Escobar theorem for continuous domains and a new pullback theorem for computable embeddings.

As an application we adapt the classical notion of learning from text to computable structure theory. Our main result is a model-theoretic characterization of the learnability from text for classes of structures. We show that a family of structures is learnable from text if and only if the structures can be distinguished in terms of their theories restricted to positive infinitary Σ_2 sentences. The methods combine the approaches of algorithmic learning theory and computable structure theory.

This is a joint work with N. Bazhenov, E. Fokina, D. Rosseger and S. Vatev.

Multilanguage Dynamic Wordnet: Timeflow Hydra

Borislav Rizov, Sofia University

Hydra is a Wordnet management system that integrates synsets from multiple languages into a shared relational structure, modeled as a Kripke frame. It includes an intuitive graphical user interface (GUI) for searching, editing, and aligning language objects. Data retrieval is supported by a modal logic query language, enabling flexible interaction with multilingual Wordnets. The original Hydra system, like other existing Wordnet platforms, captures only the current state of the data, leaving open important questions about how Wordnet structure and consistency evolve over time. To address this limitation, Time Flow Hydra introduces a dynamic Wordnet model with a discrete temporal embedding. This model stores all historical states of Wordnet objects and allows users to access them simultaneously.

Time Flow Hydra manages data from multiple languages within a unified relational structure (a Kripke frame) and enables users to query the data at any chosen point in time. It also provides fine-grained access control through a robust permission framework, ensuring that users can manage and interact with the data securely. With Time Flow Hydra, users can track changes over time and analyze the effects of data evolution, both desired and undesired. For example, one can query which synsets had two hyponyms 10 days ago and three hyponyms 5 days later.

Learning Rational Probability Distributions with Bisequential Variational Autoencoders

Georgi Shopov, Sofia University

Recurrent neural networks (RNNs) are widely used in machine learning for modelling probability distributions over finite sequences. They process input sequences from left to right by deterministically transitioning between states, a behavior analogous to sequential transducers. Consequently, RNNs and sequential transducers exhibit the same representational limitations: their fixed left-to-right causal structure prevents them from capturing the full class of rational probability distributions, that is, those expressible by nondeterministic transducers.

Bisequential decompositions overcome those limitations by using a co-sequential transducer to scan the input from right to left and annotate each position with a latent co-regular feature, followed by a sequential transducer that processes the resulting annotated sequence from left to right to compute its probability. Such decompositions represent exactly the class of rational probability distributions.

Building on this insight, we introduce bisequential variational autoencoders, a neural architecture based on classical variational autoencoders in which a right-to-left RNN computes the latent encoding and a left-to-right RNN – the latent decoding. We derive a corresponding evidence lower bound for this architecture and show how standard variational optimisation can be used to train such models. Finally, we establish that bisequential variational autoencoders represent exactly the class of rational probability distributions, whence, they are strictly more expressive than standard left-to-right RNNs.

Are Periods $\mathcal{M}^2$-Computable Real Numbers?

Dimitar Chaltakov and Ivan Georgiev, Sofia University

We are interested in the subrecursive computability of those complex numbers called periods and popularized in 2001 by Maxim Kontsevich and Don Zagier as the complex numbers whose real and imaginary parts are values of absolutely convergent integral of rational functions with rational coefficients, over domains in $\mathbb{R}^n$ given by polynomial inequalities with rational coefficients.

The particular classes of functions in question are $\mathcal{M}^2 \subseteq \mathcal{L}^2 \subseteq \mathcal{E}^2 \subset \mathcal{E}^3$. In 2008 Masahiko Yoshinaga proved that the real periods are elementary (i.e. $\mathcal{E}^3$-computable). In 2009 Katrin Tent and Martin Ziegler proved that periods are lower elementary (i.e. $\mathcal{L}^2$-computable). We reason about the question whether periods are $\mathcal{M}^2$-computable. Having in mind the theorem about the complexity of integration in case of $\mathcal{M}^2$ (Ivan Georgiev, 2020), it turns out that the $\mathcal{M}^2$-computability of the periods rely on two logically independent questions. For the sake of brevity we call them: the requirement about the $\mathcal{M}^2$-computability of the integrand and the requirement about the analytic continuations of the integrand. The first question is solved, but the second requires additional work.

AI that thinks in its mind

Dimiter Dobrev, IMI, Bulgarian Academy of Sciences

More and more scientists are stating that the statistical approach known as large language models (LLM) is not sufficient to create a thinking machine. The opinion prevails that if we want the machine to think, it must understand the world, or more precisely, search for a world model. (Just a few days ago, Yann LeCun, one of the leading specialists in the field of AI, joined this opinion.) Let's ask ourselves the question, what is a world model? It is a function from $\mathbb{N}$ to $\mathbb{N}$. In mathematical logic we deal with the description of such functions. The most famous class of such functions are computable functions, which we describe with programming languages. We will need to extend the class of computable functions by adding semi-decidable predicates and oracles. We will create a language for description of worlds that will allow automatic search for a world model. For this purpose, the language will describe separate dependencies and the resulting description will be a sum of these separate dependencies. The basis of the language for description of worlds will be Event-Driven models. We will have real Event-Driven models that partition the set of states of the world into disjoint subsets. In addition to the real ones, we will also have abstract Event-Driven models. Using the language for description of worlds, we will describe the world of the chess game. We have already done this in a previous paper, but then the agent could see the board, and now he will play blind. When you do not see the board, the task is more complicated and requires the addition of abstract Event-Driven models. The result will be a world model through which AI will be able to think in its mind and plan its actions.

Levitz class and punctually non-standard models of natural numbers

Ivan Georgiev, Sofia University

We consider punctual copies of $(\mathbb{N}, S)$: these are primitive recursive structures with domain $\mathbb{N}$, which are isomorphic to the natural numbers with successor. In this talk we will discuss how to produce such punctual copies, relative to which the class of primitive recursive functions is not standard. We will present an ad-hoc construction in which the sum and the product are primitive recursive, but all functions of exponential growth are not primitive recursive.

In order to handle the exponential functions, we introduce the Levitz class and we will prove that normal forms for the functions in the class can be computed in a primitive recursive way. Then we present the technique of islands and archipelago to produce a structure, in which the sum, the product and all exponential functions with constant base are primitive recursive, but the predecessor function is not primitive recursive.

Representations of real numbers in irrational bases

Lars Kristiansen, University of Oslo, Norway

The past 10 years, I have been working on degrees of representations of irrational numbers. Together with a number of coauthors I have developed a degree structure which we now refer to as the S-degrees. I will briefly explain how the theory leading up to the S-degrees can be generalised such that we also can deal with

– representations of all the real numbers (not only the irrationals) and representations of other sets of continuum cardinality

– representations of a non-computable nature, e.g., Cauchy sequences without modulus of convergence.

These generalisations certainly open for a study of a wide range of representations, but it remains to be seen to if our proof techniques will be fruitful when we apply them in this generalised setting (it remains to be seen if we can prove any interesting theorems).

I will, in particular, draw attention to some representations (of real numbers) based on irrational numbers related to the golden ratio. We might be able to prove some interesting theorem about such representation, and we might not, anyway, this is a potentially entertaining topic which I assume will be of general interest.

On Some Elementary Theories for Rotation in the Line-Based Euclidean Plane II: The Unoriented Irrational Angle

Lyuba Konova and Tinko Tinchev, Sofia University

In this report we continue the study of the topic "On some elementary theories for rotation in the line-based Euclidean plane". Our main interest is in the sentences that are true in the structure of lines in the Euclidean plane, in a language with a predicate expressing whether the angle between two lines is fixed. The problem has four major cases, depending on whether the angle is oriented and whether it is a rational multiple of π. In the present report we provide a suitable axiomatization for the case when the angle is an irrational multiple of π and is unoriented. We prove both the completeness of the theory thus constructed and that its decision problem lies in the complexity class $PSPACE$.

Lower bounds on Boolean and modal circuit-size

Petar Iliev, IMI, Bulgarian Academy of Sciences

In the first part of the talk, we are going to explain why proving lower bounds on the size of Boolean circuits and formulae computing explicit Boolean functions would result in the separation of the classes P and NP, and how the initial optimism in this direction was partly inspired by the classical theorems of Shannon, Subotovskaya, Khrapchenko, and Razborov. We conclude this part with a brief summary of the fundamental difficulties in the field encapsulated by the Razborov-Rudich theorem which, over-simplifying a bit, states that all known proofs of lower bounds on the circuit-size of explicit Boolean functions possess certain property, but no argument with this property would lead to the separation of P and NP.

When encountering fundamental difficulties like those described by Razborov and Rudich, efforts are usually redirected to the flanks where it is possible to obtain meaningful results. Recently, it has become clear that modal propositional logics (interpreted broadly as including various temporal, epistemic, and intuitionistic systems) offer numerous questions concerning the minimal size of modal formulae and circuits expressing properties of Kripke models. Some representative results and related unresolved problems in this line of work form the second part of the talk.

PM comme un problème mathématique

Stefan Gerdjikov, Sofia University

Consider a binary cube $\{0,1\}^n$ and pick 3 of its vertices uniformly at random. What is the probability for the resulting, possibly degenerate, triangle to be equilateral? And what is the asymptotic of this probability as n tends to infinity?

Maybe only few people, most of whom are readers of the *Revue de la filière mathématique RMS* from 2024, care about these questions. Yet, if you leave aside the obvious combinatorics and abandon the self-offering first-year calculus, it turns out that the problem relates the continuous with the discrete world in an unexpectedly beautiful way.

In this talk, we shall recall the so-called Local Limit Theorem that provides an off-the-shelf powerful technique to solve problems of these kind. Then we shall generalise the problem from equilateral triangles to equidistant d points on the cube or even originating from a more involved distribution. This will lead us to a more careful study of lattices, graph structure of certain matrices and Hamel bases in order to get a better understanding in this otherwise probabilistically-analytic problem.

This talk is based on a joint work with Martin Michev and Mladen Savov.

Markovianity of Language Models: Theory and a Polynomial-time Decision Procedure

Stoyan Mihov, IICT, Bulgarian Academy of Sciences

Classical k-gram language models are, by construction, fixed-order Markov processes, whereas modern neural language models (RNN-based, Transformer-based, etc.) are not known to satisfy any finite-order Markov property. Determining whether a given language model satisfies the Markov property – and, if so, what its minimal order is – has important implications for model compression, sampling algorithms, interpretability, and theoretical analyses of long-range dependencies.

In this talk we adopt the viewpoint of stochastic automata theory: any language model that runs on a digital computer can be represented as a stochastic finite-state transducer (SFST). Using classic results on minimality and equivalence of stochastic automata, we prove that Markovianity is a decidable property for such models.

Building on this decidability proof, we construct a polynomial-time decision algorithm that (i) decides whether the SFST defines a k-order Markov process for some finite k, and (ii) returns the smallest such k when the answer is affirmative. The algorithm runs in time polynomial in the number of states of the transducer.

The fragment of elementary plane Euclidean geometry based on perpendicularity alone with complexity PSPACE-complete

Tatyana Ivanova, IMI, Bulgarian Academy of Sciences and Tinko Tinchev, Sofia University

A. Tarski uses in his system for elementary Euclidean geometry only the primitive concept of point, and the two primitive relations betweenness and equidistance. Another approach is for the relations to be on lines instead of points. W. Schwabhäuser and L. Szczerba showed that perpendicularity together with the ternary relation of co-punctuality are sufficient for dimension two, i.e. they may be used as a system of primitive relations for elementary plane Euclidean geometry. In this talk we give a complete axiomatization for the fragment of elementary plane Euclidean geometry based on perpendicularity alone. We show that this theory is not finitely axiomatizable, that it is decidable and that the complexity is PSPACE-complete. In contrast the complexity of elementary plane Euclidean geometry is exponential.