вариант	ф. номер	група	поток	курс	специалност
Вариант 1					**КН**
Име:					

Изпит по СЕП
15 февруари 2022 г.

Зад. 1. Нека $\mathcal{A}$ е област на Скот. Докажете, че всяко изображение $f \in [\mathcal{A} \overset{\text{н}}{\to} \mathcal{A}]$ притежава най-малка неподвижна точка.

Зад. 2. Нека с $\mathcal{F}_1$ да означим областта на Скот от едноместните частични функции в естествените числа.

а) Дайте пример за изображение $\Gamma_1 : \mathcal{F}_1 \to \mathcal{F}_1$, което има неподвижна точка, която е тотална функция, но най-малката неподвижна точка на Γ_1 не е тотална.

б) Дайте пример за изображение $\Gamma_2 : \mathcal{F}_1 \to \mathcal{F}_1$, който има безкрайно много неподвижни точки.

в) Определете дали Вашите примери за Γ_1 и Γ_2 са непрекъснати изображения.

Зад. 3. Дефинираме релацията $\lhd_{\texttt{a}} \subseteq [\![\texttt{a}]\!] \times \texttt{PCF}_{\texttt{a}}$ като:

$$n \lhd_{\texttt{nat}} \tau \overset{\text{деф}}{\iff} (n \neq \bot^{[\![\texttt{nat}]\!]} \implies \tau \Downarrow_{\texttt{nat}} \texttt{n}).$$

$$f \lhd_{\texttt{b}\to\texttt{c}} \tau \overset{\text{деф}}{\iff} (\forall e \in [\![\texttt{b}]\!])(\forall \mu \in \texttt{PCF}_{\texttt{b}})[\ e \lhd_{\texttt{b}} \mu \implies f(e) \lhd_{\texttt{c}} \tau(\mu)\ ].$$

Да разгледаме произволен тип $\texttt{a}$ и произволен терм τ. Нека $(d_i)_{i=0}^{\infty}$ е верига, за която $d_i \lhd_{\texttt{a}} \tau$. Докажете, че $\bigsqcup_i d_i \lhd_{\texttt{a}} \tau$.

Зад. 4. Да дефинираме релацията $\Gamma \vdash \tau : \texttt{a}$ по следния начин:

$$\frac{}{\Gamma \vdash \texttt{n} : \texttt{nat}}\ \text{(const)}$$

$$\frac{\texttt{x} \in \texttt{dom}(\Gamma) \qquad \Gamma(\texttt{x}) = \texttt{a}}{\Gamma \vdash \texttt{x} : \texttt{a}}\ \text{(var)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{nat}}{\Gamma \vdash \tau_1 + \tau_2 : \texttt{nat}}\ \text{(plus)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{nat}}{\Gamma \vdash \tau_1 - \tau_2 : \texttt{nat}}\ \text{(minus)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{nat}}{\Gamma \vdash \tau_1 \texttt{==} \tau_2 : \texttt{nat}}\ \text{(eq)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{a} \qquad \Gamma \vdash \tau_3 : \texttt{a}}{\Gamma \vdash \texttt{if}\ \tau_1\ \texttt{then}\ \tau_2\ \texttt{else}\ \tau_3 : \texttt{a}}\ \text{(if)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{a} \to \texttt{b} \qquad \Gamma \vdash \tau_2 : \texttt{a}}{\Gamma \vdash \tau_1 \tau_2 : \texttt{b}}\ \text{(app)}$$

$$\frac{\Gamma \vdash \tau : \texttt{a} \to \texttt{a}}{\Gamma \vdash \texttt{fix}(\tau) : \texttt{a}}\ \text{(fix)}$$

$$\frac{\texttt{x} \notin \texttt{dom}(\Gamma) \qquad \Gamma, \texttt{x} : \texttt{a} \vdash \tau : \texttt{b}}{\Gamma \vdash \lambda \texttt{x} : \texttt{a}\ .\ \tau : \texttt{a} \to \texttt{b}}\ \text{(lambda)}$$

Докажете, че имаме извода:

$$\frac{\Gamma \vdash \rho : \texttt{a} \qquad \texttt{x} \notin \texttt{dom}(\Gamma) \qquad \Gamma, \texttt{x} : \texttt{a} \vdash \tau}{\Gamma \vdash \tau[\texttt{x}/\rho]}$$

вариант	ф. номер	група	поток	курс	специалност
Вариант 2					**КН**
Име:					

Изпит по СЕП
15 февруари 2022 г.

Зад. 1. Нека $\mathcal{A}$ е област на Скот. Докажете, че всяко изображение $f \in [\mathcal{A} \overset{\text{н}}{\to} \mathcal{A}]$ притежава най-малка неподвижна точка.

Зад. 2. Нека с $\mathcal{F}_1$ да означим областта на Скот от едноместните частични функции в естествените числа.

а) Дайте пример за изображение $\Gamma_1 : \mathcal{F}_1 \to \mathcal{F}_1$, което има неподвижна точка, която е тотална функция, но най-малката неподвижна точка на Γ_1 не е тотална.

б) Дайте пример за изображение $\Gamma_2 : \mathcal{F}_1 \to \mathcal{F}_1$, който има безкрайно много неподвижни точки.

в) Определете дали Вашите примери за Γ_1 и Γ_2 са непрекъснати изображения.

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$$f \lhd_{\texttt{b}\to\texttt{c}} \tau \overset{\text{деф}}{\iff} (\forall e \in [\![\texttt{b}]\!])(\forall \mu \in \texttt{PCF}_{\texttt{b}})[\ e \lhd_{\texttt{b}} \mu \implies f(e) \lhd_{\texttt{c}} \tau(\mu)\ ].$$

Да разгледаме произволен тип $\texttt{a}$ и произволен терм τ. Нека $(d_i)_{i=0}^{\infty}$ е верига, за която $d_i \lhd_{\texttt{a}} \tau$. Докажете, че $\bigsqcup_i d_i \lhd_{\texttt{a}} \tau$.

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$$\frac{\texttt{x} \in \texttt{dom}(\Gamma) \qquad \Gamma(\texttt{x}) = \texttt{a}}{\Gamma \vdash \texttt{x} : \texttt{a}}\ \text{(var)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{nat}}{\Gamma \vdash \tau_1 + \tau_2 : \texttt{nat}}\ \text{(plus)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{nat}}{\Gamma \vdash \tau_1 - \tau_2 : \texttt{nat}}\ \text{(minus)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{nat}}{\Gamma \vdash \tau_1 \texttt{==} \tau_2 : \texttt{nat}}\ \text{(eq)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{nat} \qquad \Gamma \vdash \tau_2 : \texttt{a} \qquad \Gamma \vdash \tau_3 : \texttt{a}}{\Gamma \vdash \texttt{if}\ \tau_1\ \texttt{then}\ \tau_2\ \texttt{else}\ \tau_3 : \texttt{a}}\ \text{(if)}$$

$$\frac{\Gamma \vdash \tau_1 : \texttt{a} \to \texttt{b} \qquad \Gamma \vdash \tau_2 : \texttt{a}}{\Gamma \vdash \tau_1 \tau_2 : \texttt{b}}\ \text{(app)}$$

$$\frac{\Gamma \vdash \tau : \texttt{a} \to \texttt{a}}{\Gamma \vdash \texttt{fix}(\tau) : \texttt{a}}\ \text{(fix)}$$

$$\frac{\texttt{x} \notin \texttt{dom}(\Gamma) \qquad \Gamma, \texttt{x} : \texttt{a} \vdash \tau : \texttt{b}}{\Gamma \vdash \lambda \texttt{x} : \texttt{a}\ .\ \tau : \texttt{a} \to \texttt{b}}\ \text{(lambda)}$$

Докажете, че имаме извода:

$$\frac{\Gamma \vdash \rho : \texttt{a} \qquad \texttt{x} \notin \texttt{dom}(\Gamma) \qquad \Gamma, \texttt{x} : \texttt{a} \vdash \tau}{\Gamma \vdash \tau[\texttt{x}/\rho]}$$