Lopez-Escobar theorem for continuous domains and Learning theory

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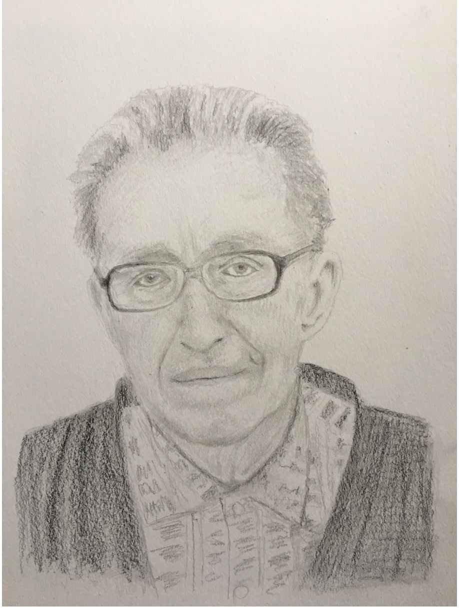
Dedicated to the memory of my teacher Prof. Skordev



Introduction

A common theme in the study of countable mathematical structures is that isomorphism invariant properties have syntactic characterizations.
Examples:

- the existence of Scott sentences for countable structures [Scott 63],
- the Lopez-Escobar theorem, which says that every invariant Borel subset in the space of countable structures is definable in the infinitary logic $L_{\omega_1\omega}$ [Lopez-Escobar 65; Vaught 74],
- a relation on a structure that is Σ^0_α in every copy is definable by a computable Σ_α $L_{\omega_1\omega}$-formula [Ash, Knight, Manasse, Sleman 89; Chisholm 90].

The Borel hierarchy for non-metrizable spaces

Definition (Selivanov 2006)

Let (X, τ) be a topological space. For each countable ordinal $\alpha \geq 1$ we define $\mathbf{\Sigma}^0_\alpha(X, \tau)$ inductively as follows.

1. $\mathbf{\Sigma}^0_1(X, \tau) = \tau$ - the open sets.
2. For $\alpha > 1$, $\mathbf{\Sigma}^0_\alpha(X, \tau)$ is the set of all subsets A of X which can be expressed in the form

$$A = \bigcup_{i \in \omega} B_i \setminus B'_i,$$

   where B_i and B'_i are in $\mathbf{\Sigma}^0_{\beta_i}(X, \tau)$ for some $\beta_i < \alpha$, for each i.

Let $\mathbf{\Pi}^0_\alpha(X, \tau) = \{X \setminus A \mid A \in \mathbf{\Sigma}^0_\alpha(X, \tau)\}$.
Define $B(X, \tau) = \bigcup_{\alpha < \omega_1} \mathbf{\Sigma}^0_\alpha(X, \tau)$ to be the Borel subsets of (X, τ).

For metrizable spaces this is equivalent to the standard Borel hierarchy.

Scott topology

Let $(P, \leq)$ be a partial order.

- A set $D \subseteq P$ is directed if $D \neq \emptyset$ and for all $a, b \in D$, there is $d \in D$ with $a \leq d$ and $b \leq d$.
- A partial order P is a directed complete partial order (dcpo) if every directed $D \subseteq P$ has a supremum $\sup D$ in P.
- A set U is Scott open if U is an upper set, i.e., $x \in U$ and $x \leq y \implies y \in U$, and for every directed set D with $\sup D \in U$ we have that $D \cap U \neq \emptyset$. The Scott open sets of P form a topology on P, the Scott topology.

Example

Consider the Scott topology on 2^ω equipped with the dcpo given by $f \subseteq g$ if $f(i) = 1 \implies g(i) = 1$. It has a natural countable basis given by the basic open sets

$$2^\omega, \emptyset, \text{ and } O_n = \{f \in 2^\omega \mid f(n) = 1\} \text{ for all } n \in \omega.$$

Cantor topology and Scott topology on $\mathrm{Mod}(\tau)$

Fix a countable relational signature τ. Here we consider only τ-structures $\mathcal{A}$ with domain ω.
We fix an encoding of (atomic diagrams of) τ-structures. This allows us to identify τ-structures with elements of the Cantor space 2^ω. More formally, we talk about the space of τ-models $\mathrm{Mod}(\tau)$ which is homeomorphic to 2^ω.

Consider a new topological space $\mathrm{Mod_p}(\tau)$. Let τ contains $=$ and $\neq$. The elements of $\mathrm{Mod_p}(\tau)$ are still τ-structures with domain ω, but the space is equipped with the Scott topology.
Let F be a non-empty finite set of atomic formulas (no negation). Then the basic open set $\mathrm{U_F}$ contains all structures $\mathcal{A}$ satisfying $\wedge\mathrm{F}$.

There is a natural dcpo on $\mathrm{Mod_p}(\tau)$ given by

$$\mathcal{A} \preceq \mathcal{B} \iff \forall \mathrm{R} \in \tau(\mathrm{R}^{\mathcal{A}} \subseteq \mathrm{R}^{\mathcal{B}}).$$

$L_{\omega_1\omega}$ formulas

Let I be a countable set.

- The Σ_0 formulas (Π_0 formulas) are quantifier free τ-formulas. For $\alpha \geq 1$:
- $\varphi(\bar{u})$ is Σ_α formula if it has the form

$$\varphi(\bar{u}) = \bigvee_{i \in I} \exists \bar{x}_i (\phi_i(\bar{u}, \bar{x}_i) \wedge \psi_i(\bar{u}, \bar{x}_i)),$$

  where $\phi_i(\bar{u}, \bar{x}_i)$ is a Σ_{β_i} and $\psi_i(\bar{u}, \bar{x}_i)$ is Π_{β_i}, for some $\beta_i < \alpha$.
- $\varphi(\bar{u})$ is Π_α formula if it has the form

$$\varphi(\bar{u}) = \bigwedge_{i \in I} \forall \bar{x}_i (\phi_i(\bar{u}, \bar{x}_i) \vee \psi_i(\bar{u}, \bar{x}_i)),$$

  where $\phi_i(\bar{u}, \bar{x}_i)$ is a Σ_{β_i} and $\psi_i(\bar{u}, \bar{x}_i)$ is a Π_{β_i}, for some $\beta_i < \alpha$.

The Lopez-Escobar theorem

In the classical setting the Lopez-Escobar theorem establishes a correspondence between subsets of $\mathrm{Mod}(\tau)$ defined by sentences in the infinitary logic $\mathrm{L}_{\omega_1\omega}$ and the Borel sets.

Theorem (Lopez-Escobar 65, Vaught 74)

Let $\mathcal{K}$ be a subclass of $\mathrm{Mod}(\tau)$ which is closed under isomorphisms. Let $\alpha > 0$ be a countable ordinal. Then $\mathcal{K}$ is $\mathbf{\Sigma}^0_\alpha$ (in the Borel hierarchy) if and only if $\mathcal{K}$ is axiomatizable by a Σ_α-sentence.

Theorem (Bazhenov, Fokina, Rossegger, S., and Vatev, 2023)

Let $\mathcal{K}$ be a subclass of $\mathrm{Mod_p}(\tau)$ which is closed under isomorphisms. Let $\alpha > 0$ be a countable ordinal. Then $\mathcal{K}$ is $\mathbf{\Sigma}^0_\alpha$ in the space $\mathrm{Mod_p}(\tau)$ if and only if $\mathcal{K}$ is axiomatizable by a $\Sigma^{\mathrm{p}}_\alpha$-sentence.

We use a forcing relation used by Soskov in 2004 in order to characterize the relatively intrinsic relations for the enumeration reducibility, but our presentation is closer to Montalbán 2021.

A hierarchy of positive infinitary formulas

Let I be a countable set.

- Let $\alpha = 0$. Then:
  - the Σ^p_0 formulas are the finite conjunctions of atomic τ-formulas.
  - the Π^p_0 formulas are the finite disjunctions of negations of atomic τ-formulas.
- Let $\alpha = 1$. Then:
  - $\varphi(\bar{u})$ is a Σ^p_1 formula if it has the form

    $$\varphi(\bar{u}) = \bigvee_{i \in I} \exists \bar{x}_i \psi_i(\bar{u}, \bar{x}_i),$$

    where for each $i \in I$, $\psi_i(\bar{u}, \bar{x}_i)$ is a Σ^p_0 formula.
  - $\varphi(\bar{u})$ is a Π^p_1 formula if it has the form

    $$\varphi(\bar{u}) = \bigwedge_{i \in I} \forall \bar{x}_i \psi_i(\bar{u}, \bar{x}_i),$$

    where for each $i \in I$, $\psi_i(\bar{u}, \bar{x}_i)$ is a Π^p_0 formula.

A hierarchy of positive infinitary formulas

- Let $\alpha \geq 2$. Then:
  - $\varphi(\bar{u})$ is Σ^p_α formula if it has the form

$$\varphi(\bar{u}) = \bigvee_{i \in I} \exists \bar{x}_i(\phi_i(\bar{u}, \bar{x}_i) \wedge \psi_i(\bar{u}, \bar{x}_i)),$$

    where $\phi_i(\bar{u}, \bar{x}_i)$ is a $\Sigma^p_{\beta_i}$ and $\psi_i(\bar{u}, \bar{x}_i)$ is $\Pi^p_{\beta_i}$, for some $\beta_i < \alpha$.
  - $\varphi(\bar{u})$ is Π^p_α formula if it has the form

$$\varphi(\bar{u}) = \bigwedge_{i \in I} \forall \bar{x}_i(\phi_i(\bar{u}, \bar{x}_i) \vee \psi_i(\bar{u}, \bar{x}_i)),$$

    where $\phi_i(\bar{u}, \bar{x}_i)$ is a $\Sigma^p_{\beta_i}$ and $\psi_i(\bar{u}, \bar{x}_i)$ is a $\Pi^p_{\beta_i}$, for some $\beta_i < \alpha$.

Effective version: computable Σ^p_α (Π^p_α) formulas.

Computable embeddings

Knight, S. Miller, and Vander Boom, 2007:

Let $\mathcal{K}_0$ be a class of τ_0-structures, and $\mathcal{K}_1$ be a class of τ_1-structures.

Definition

A Turing operator $\Phi = \varphi_e$ is a Turing computable embedding of $\mathcal{K}_0$ into $\mathcal{K}_1$, denoted by $\Phi\colon \mathcal{K}_0 \leq_{tc} \mathcal{K}_1$, if Φ satisfies the following:

1. For any $\mathcal{A} \in \mathcal{K}_0$, the function $\varphi_e^{D(\mathcal{A})}$ is the characteristic function of the atomic diagram of a structure from $\mathcal{K}_1$. This structure is denoted by $\Phi(\mathcal{A})$.
2. For any $\mathcal{A}, \mathcal{B} \in \mathcal{K}_0$ we have: $\mathcal{A} \cong \mathcal{B}$ if and only if $\Phi(\mathcal{A}) \cong \Phi(\mathcal{B})$.

Definition

An enumeration operator Γ is a computable embedding of $\mathcal{K}_0$ into $\mathcal{K}_1$, denoted by $\Gamma\colon \mathcal{K}_0 \leq_c \mathcal{K}_1$, if Γ satisfies the following:

1. For $\mathcal{A} \in \mathcal{K}_0$, $\Gamma(\mathcal{A})$ is the (positive) atomic diagram of a structure from $\mathcal{K}_1$.
2. For any $\mathcal{A}, \mathcal{B} \in \mathcal{K}_0$ we have: $\mathcal{A} \cong \mathcal{B}$ if and only if $\Gamma(\mathcal{A}) \cong \Gamma(\mathcal{B})$.

A pullback theorem for computable embeddings

Computable embeddings have the useful property of monotonicity: If $\Gamma\colon \mathcal{K}_0 \leq_{\mathrm{c}} \mathcal{K}_1$ and $\mathcal{A} \subseteq \mathcal{B}$ are structures from $\mathcal{K}_0$, then we have $\Gamma(\mathcal{A}) \subseteq \Gamma(\mathcal{B})$.

Turing computable embeddings do in general not have the monotonicity properties of computable embeddings.

Proposition (Greenberg; Kalimullin)

If $\mathcal{K}_0 \leq_{\mathrm{c}} \mathcal{K}_1$, then $\mathcal{K}_0 \leq_{\mathrm{tc}} \mathcal{K}_1$. The converse is not true.

Theorem (Pullback Theorem)

[Bazhenov, Fokina, Rossegger, S., and Vatev, 2023] Let $\mathcal{K} \subseteq \mathrm{Mod}_{\mathrm{p}}(\tau)$ and $\mathcal{K}' \subseteq \mathrm{Mod}_{\mathrm{p}}(\tau')$ be closed under isomorphism. Let $\Gamma_{\mathrm{e}} : \mathcal{K} \leq_{\mathrm{c}} \mathcal{K}'$, then for any computable $\Sigma^{\mathrm{p}}_{\alpha}$ (or $\Pi^{\mathrm{p}}_{\alpha}$) τ'-sentence φ we can effectively find a computable $\Sigma^{\mathrm{p}}_{\alpha}$ (or $\Pi^{\mathrm{p}}_{\alpha}$) τ-sentence φ^* such that for all $\mathcal{A} \in \mathcal{K}$,

$$\mathcal{A} \models \varphi^* \text{ iff } \Gamma_{\mathrm{e}}(\mathcal{A}) \models \varphi.$$

Relativizing

A function $\Psi : 2^\omega \to 2^\omega$ is continuous in the Cantor topology iff there exists a Turing operator Φ_e and a set $A \in 2^\omega$ such that $\Psi(X) = \Phi_e(A \oplus X)$ for all $X \in 2^\omega$. Thus, for any two classes $\mathcal{K}_0$ and $\mathcal{K}_1$:

$$\mathcal{K}_0 \leq_{\text{Cantor}} \mathcal{K}_1 \iff \mathcal{K}_0 \leq^X_{tc} \mathcal{K}_1 \text{for some set } X.$$

Definition (Case, 71)

A set $A \subseteq \omega$ defines a generalized enumeration operator $\Gamma : 2^\omega \to 2^\omega$ iff for each set $B \subseteq \omega$,

$$\Gamma(B) = \{x \mid \langle x, v \rangle \in A \;\&\; D_v \subseteq B\}.$$

Proposition (Folklore)

. A function Γ: $2^\omega \to 2^\omega$ is continuous in the Scott topology iff Γ is a generalized enumeration operator.

A pullback theorem for Scott continuous embeddings

Definition

A continuous function Γ in the Scott topology is a continuous embedding of $\mathcal{K}_0$ into $\mathcal{K}_1$, denoted by $\mathcal{K}_0 \leq_{\text{Scott}} \mathcal{K}_1$ if Γ satisfies the following:

1. For $\mathcal{A} \in \mathcal{K}_0$, $\Gamma(\mathcal{A})$ is the (positive) atomic diagram of a structure from $\mathcal{K}_1$.
2. For any $\mathcal{A}, \mathcal{B} \in \mathcal{K}_0$, we have $\mathcal{A} \cong \mathcal{B}$ if and only if $\Gamma(\mathcal{A}) \cong \Gamma(\mathcal{B})$.

Theorem (Pullback Theorem)

[Bazhenov, Fokina, Rossegger, S., and Vatev, 2023] Let $\mathcal{K} \subseteq \text{Mod}_{\text{p}}(\tau)$ and $\mathcal{K}' \subseteq \text{Mod}_{\text{p}}(\tau')$ be closed under isomorphism and $\alpha > 0$ be a countable ordinal. Let Γ be a Scott-continuous embedding from $\mathcal{K}$ into $\mathcal{K}'$ ($\mathcal{K} \leq_{\text{Scott}} \mathcal{K}'$). Then for any $\Sigma^{\text{p}}_{\alpha}$ (or Π^{p}_{α}) τ'-sentence φ we can find a $\Sigma^{\text{p}}_{\alpha}$ (or Π^{p}_{α}) τ-sentence φ^* such that for all $\mathcal{A} \in \mathcal{K}$,

$$\mathcal{A} \models \varphi^* \text{ iff } \Gamma(\mathcal{A}) \models \varphi.$$

Learning for families of algebraic structures

- Fix a computable signature τ. Let $\mathcal{K}$ be a countable family of countable τ-structures.
- Step-by-step, we obtain larger and larger finite pieces of a τ-structure $\mathcal{A}$. In addition, we assume that this $\mathcal{A}$ is isomorphic to some structure from the class $\mathcal{K}$.

Problem: Is it possible to identify (in the limit) the isomorphism type of $\mathcal{A}$.

The problem combines the approaches of algorithmic learning theory and computable structure theory:
We want to learn the family $\mathcal{K}$ up to isomorphism.

Learning structures from informant

Consider a family of τ-structures $\mathcal{K} = \{\mathcal{A}_i\}_{i \in \omega}$. We assume that the structures $\mathcal{A}_i$ are pairwise not isomorphic.

- The learning domain:

$$\mathrm{LD}(\mathcal{K}) = \{\mathcal{B} \mid \mathcal{B} \cong \mathcal{A}_i \text{ for some } i \in \omega; \text{ and } \mathrm{dom}(\mathcal{B}) = \omega\}$$

  The learning domain can be treated as a set of reals (i.e., $\mathrm{LD}(\mathcal{K}) \subseteq 2^\omega$).

- A learner M sees (stage by stage) finite pieces of data about a given structure from $\mathrm{LD}(\mathcal{K})$, and M outputs conjectures. More formally,

$$\text{M is a function from } 2^{<\omega} \text{ to } \omega.$$

  If $M(\sigma) = i$, then this means: "the finite string σ looks like an isomorphic copy of $\mathcal{A}_i$".

Learning structures from informant

- The learning is successful if: for every $\mathcal{B} \in \mathrm{LD}(\mathcal{K})$, if $\mathcal{B}$ is an isomorphic copy of $\mathcal{A}_i$, then

$$\lim_{k \to \infty} M(\mathcal{B} \upharpoonright k) = i.$$

Definition

The family $\mathcal{K}$ is Inf-learnable (up to isomorphism) if there exists a learner M that successfully learns the family $\mathcal{K}$.

Remark. More formally, the family $\mathcal{K}$ is $\mathrm{InfEx}_{\cong}$-learnable:

- Inf means learning from informant;
- Ex means "explanatory".

A syntactic characterization of Inf-learnability

Theorem (Bazhenov, Fokina, and San Mauro, 2020)

The following conditions are equivalent:

- The family $\mathcal{K}$ is Inf-learnable.
- There are Σ_2-sentences ψ_i, $\mathrm{i} \in \omega$ such that

$$\mathcal{A}_\mathrm{i} \models \psi_\mathrm{j} \text{ if and only if } \mathrm{i} = \mathrm{j}.$$

If ω denotes the standard ordering of natural numbers, and ω^* denotes the standard ordering of negative integers then:

Example

The pair of linear orders $\{\omega, \omega^*\}$ is learnable from informant.
Key observation:
These orders are "separated" by Σ_2-sentences: (ω has a least element) vs. (ω^* has a greatest element).

A descriptive set-theoretic characterization of Inf-learning

One of the benchmark Borel equivalence relations on the Cantor space 2^ω is the relation E_0 (almost equality).

$$\alpha \mathrm{E}_0 \beta \iff (\exists \mathrm{n})(\forall \mathrm{m} \geq \mathrm{n})(\alpha(\mathrm{m}) = \beta(\mathrm{m})).$$

Theorem (Bazhenov, Cipriani, and San Mauro, 2023)

The following conditions are equivalent:

- The family $\mathcal{K}$ is Inf-learnable.
- There is a continuous function $\Gamma : 2^\omega \to 2^\omega$ such that for all $\mathcal{A}, \mathcal{B} \in \mathrm{LD}(\mathcal{K})$ we have:

$$\mathcal{A} \cong \mathcal{B} \iff \Gamma(\mathcal{A}) \mathrm{E}_0 \Gamma(\mathcal{B}).$$

Learning structures from text

We assume that the relational signature τ contains both $=$ (equality) and $\neq$ (inequality).

[Notice that then the empty set is definable in $\mathcal{A}$ by an atomic formula $(x \neq x)$.]

We want to learn a countable family $\mathcal{K} = \{\mathcal{A}_i\}_{i \in \omega}$. Now we learn from text, i.e., from the positive information about an input structure $\mathcal{A}$.

Definition

Let $\mathcal{A}$ be a τ-structure with domain ω. A text t for the structure $\mathcal{A}$ is an arbitrary sequence $\{t(i)\}_{i \in \omega}$ such that:

- for each $i \in \omega$, $t(i)$ is an atomic formula, i.e., a formula of the form $R(a_1, \ldots, a_n)$, where $R \in \tau$ and $a_1, \ldots, a_n \in \omega$;
- the set $\{t(i) \mid i \in \omega\}$ contains precisely all atomic formulas which are true in the structure $\mathcal{A}$.

Using an encoding, texts are elements of the Baire space ω^ω.

Learning structures from text

A learner M is a function from $\omega^{<\omega}$ to ω.

Definition

Txt-learning is successful if every $\mathcal{A}$ with $\mathsf{dom}(\mathcal{A}) = \omega$ satisfies the following: if $\mathcal{A}$ is an isomorphic copy of $\mathcal{A}_i$, then for any text t for the structure $\mathcal{A}$, we have

$$\lim_{k \to \infty} M(t \upharpoonright k) = i.$$

Definition

The family $\mathcal{K}$ is Txt-learnable (up to isomorphism) if there exists a Txt-learner M that successfully learns the family $\mathcal{K}$.

Simple observations

Proposition

If $\mathcal{K}$ is Txt-learnable, then $\mathcal{K}$ is Inf-learnable.

Example

Consider the following pair of equivalence structures:

- The structure $\mathcal{A}$ has one infinite class and nothing else.
- The structure $\mathcal{B}$ has two infinite classes and nothing else.

Then the family $\mathcal{K} = \{\mathcal{A}; \mathcal{B}\}$ is Inf-learnable, but not Txt-learnable.

A syntactic characterization of Txt-learnability

Theorem (Bazhenov, Fokina, Rossegger, S., and Vatev, 2023)

The following conditions are equivalent:

- The family $\mathcal{K}$ is Txt-learnable.
- $\mathcal{K} \leq_{\mathrm{Scott}} \mathcal{K}_{\mathrm{univ}}$.
- There are Σ^{p}_2-sentences ψ_{i}, $\mathrm{i} \in \omega$ such that

$$\mathcal{A}_{\mathrm{i}} \models \psi_{\mathrm{j}} \text{ if and only if } \mathrm{i} = \mathrm{j}.$$

Idea of the proof of the theorem

Given a learner M, one can build a Scott-continuous embedding from our class $\mathcal{K}$ into some "universal learnable" class $\mathcal{K}_{\mathrm{univ}} = \{\mathcal{B}_i \mid i \in \omega\}$, where $\mathcal{B}_i = (\omega; E)$ is an equivalence structure which has infinitely many infinite classes, infinitely many classes of size $i + 1$, and nothing else.
Note that each $\mathcal{B}_i$ has its own distinguishing Σ^p_2-sentence:

$$\psi_i = \exists x_0 \dots \exists x_i [\bigwedge_{j \neq k \ j,k \leq i} (x_j \neq x_k \ \& \ x_j E x_k) \\ \& \ \forall y (\neg (y E x_0) \vee \bigvee_{l \leq i} \neg (y \neq x_l)].$$

So, $\mathcal{K} \leq_{\mathrm{Scott}} \mathcal{K}_{\mathrm{univ}}$.
Applying the Pullback theorem, we get the desired sequence of distinguishing Σ^p_2-sentences for our family $\mathcal{K}$.

Descriptive set-theoretic characterization for Txt-learnability

Consider the space $P(\omega)$ of all subsets of ω, with the Scott topology. For $X \in P(\omega)$ and $m \in \omega$, denote the m-th column of X by:

$$X^{[m]} = \{y \mid \langle m, y \rangle \in X\}.$$

The equivalence relation E_{set} is defined as follows:

$$X \, E_{set} Y \iff \{X^{[m]} \mid m \in \omega\} = \{Y^{[m]} \mid m \in \omega\}.$$

Theorem (Bazhenov, Fokina, Rossegger, S., and Vatev, 2023)

Let $\mathcal{K} = \{\mathcal{A}_i \mid i \in \omega\}$ be a family of countable τ-structures. Equivalent:

- The family $\mathcal{K}$ is Txt-learnable.
- There is a continuous function $\Gamma : Mod_p(\tau) \to P(\omega)$ such that for all $\mathcal{A}, \mathcal{B} \in LD(\mathcal{K})$:
  - $\mathcal{A} \cong \mathcal{B} \iff \Gamma(\mathcal{A}) E_{set} \Gamma(\mathcal{B})$;
  - for each $i \in \omega$ we have $\{\Gamma(A_i)^{[m]} \mid m \in \omega\} = \{\omega, \{i\}\}$.

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HAPPY ANIVERSARY PROF. VAKARELOV!