DIMITER VAKARELOV:

a friend and colleague

85th anniversary

New
and new discoveries
in deep logical waters…

May 2023
Chepyn Mountain
Distance 6+6 km
Denivelation 500 m

The Thracian Sanctuary of *Sabazios*
(2nd century B.C.)

↓

Zagreus

↓

The Greek *Dionysius* =
the Roman *Bacchus*,
the God of fertility, vitality,
and wine.

A walking stick with original Tyrolian marks,
the necessary accessory for our next pilgrim trip
to the sanctuary, this time to give a sacrifice
to Sabazios.

… savoir lire, écrire et compter.

N. Bourbaki,
Élements de mathématique,
Théorie des ensembles

MONEY, NUMBERS, & NUMERALS

Counting and Naming

How much
is 1 billion?

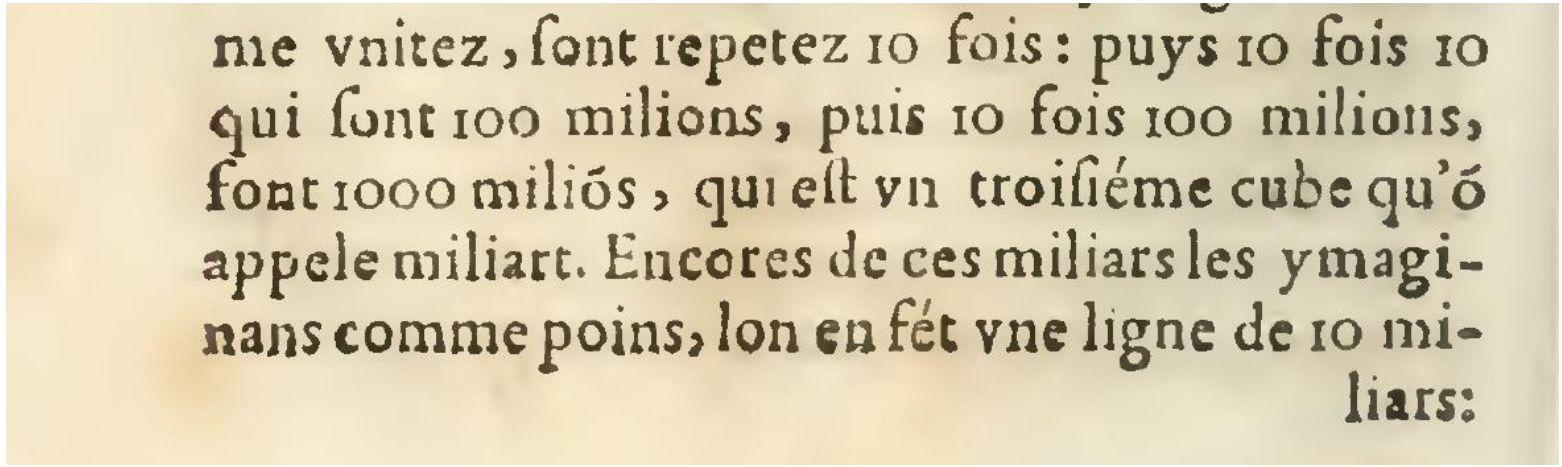
1 billion = 1000 milliard = 10^{12}

?

1 billion = 1 milliard = 10^{9}

The Origin of the Large Numerals

Milliard: Jean Trenchant, “Le Triparty en la Science des Nombres”, 1602 (*miliart*)


me vnitez, ſont repetez 10 fois: puys 10 fois 10
qui ſont 100 milions, puis 10 fois 100 milions,
font 1000 miliõs, qui eſt vn troiſiéme cube qu'õ
appele miliart. Encores de ces miliars les ymagi-
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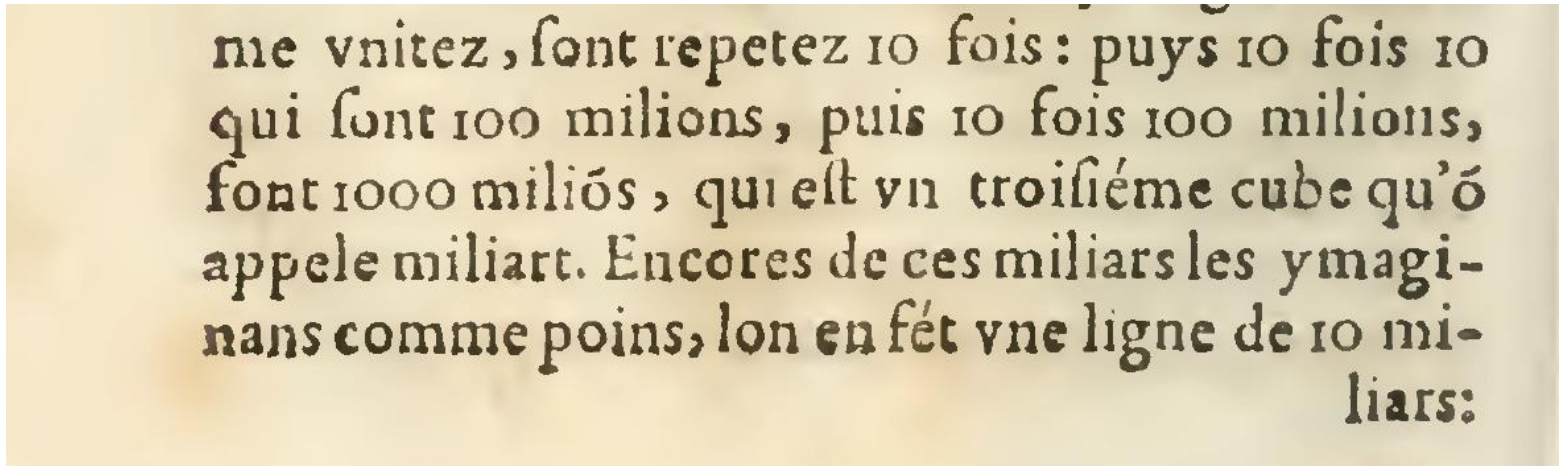
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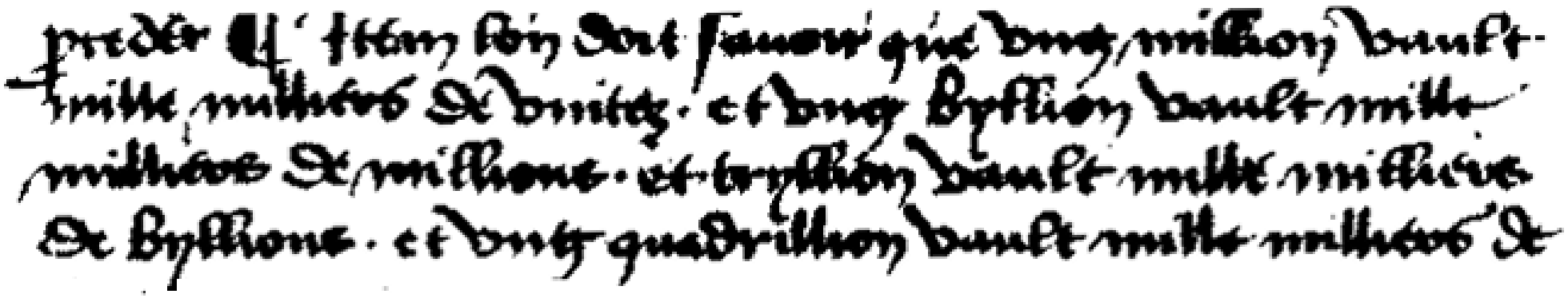
Billion, *trillion*,…: Nicolas Chuquet, 1484: “1 million = 1000.1000 [=10^6],
1 byllion = 1000.1000 millions [=10^{12}],
1 tryllion = 1000.1000 byllions, [=10^{18}]…“



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Extended long scale: Name *n*-illion = 10^{6n} ($n \geq 1$)	**LONG SCALE** (N. Chuquet, 1484) The step is 1 million = 10^6
Optional: *n*-illiard = 10^{6n+3}	1000 million = milliard = 10^9
	Billion = 10^{12}
	Billiard = 10^{15}
	Trillion = 10^{18}
	Trilliard = 10^{21}
	Quadrillion = 10^{24}

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Extended long scale: Name *n*-illion = 10^{6n} ($n \geq 1$)	**LONG SCALE** (N. Chuquet, 1484) The step is 1 million = 10^6	**SHORT SCALE** (French arithmeticians, >1800) The step is 1000
Optional: *n*-illiard = 10^{6n+3}	1000 million = milliard = 10^9	Billion = milliard = 10^9
	Billion = 10^{12}	Trillion = 10^{12}
	Billiard = 10^{15}	Quadrillion = 10^{15}
	Trillion = 10^{18}	Quintillion = 10^{18}
	Trilliard = 10^{21}	…
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THE LONG SCALE TODAY The step is 1 million = 10^6	**THE SHORT SCALE TODAY The step is 1000**
Billion = 10^{12}	Billion [= milliard] = 10^9
Trillion = 10^{18}	Trillion = 10^{12}
Quadrillion = 10^{24}	Quadrillion = 10^{15}
Quintillion = 10^{30}	Quintillion = 10^{18}
…	…

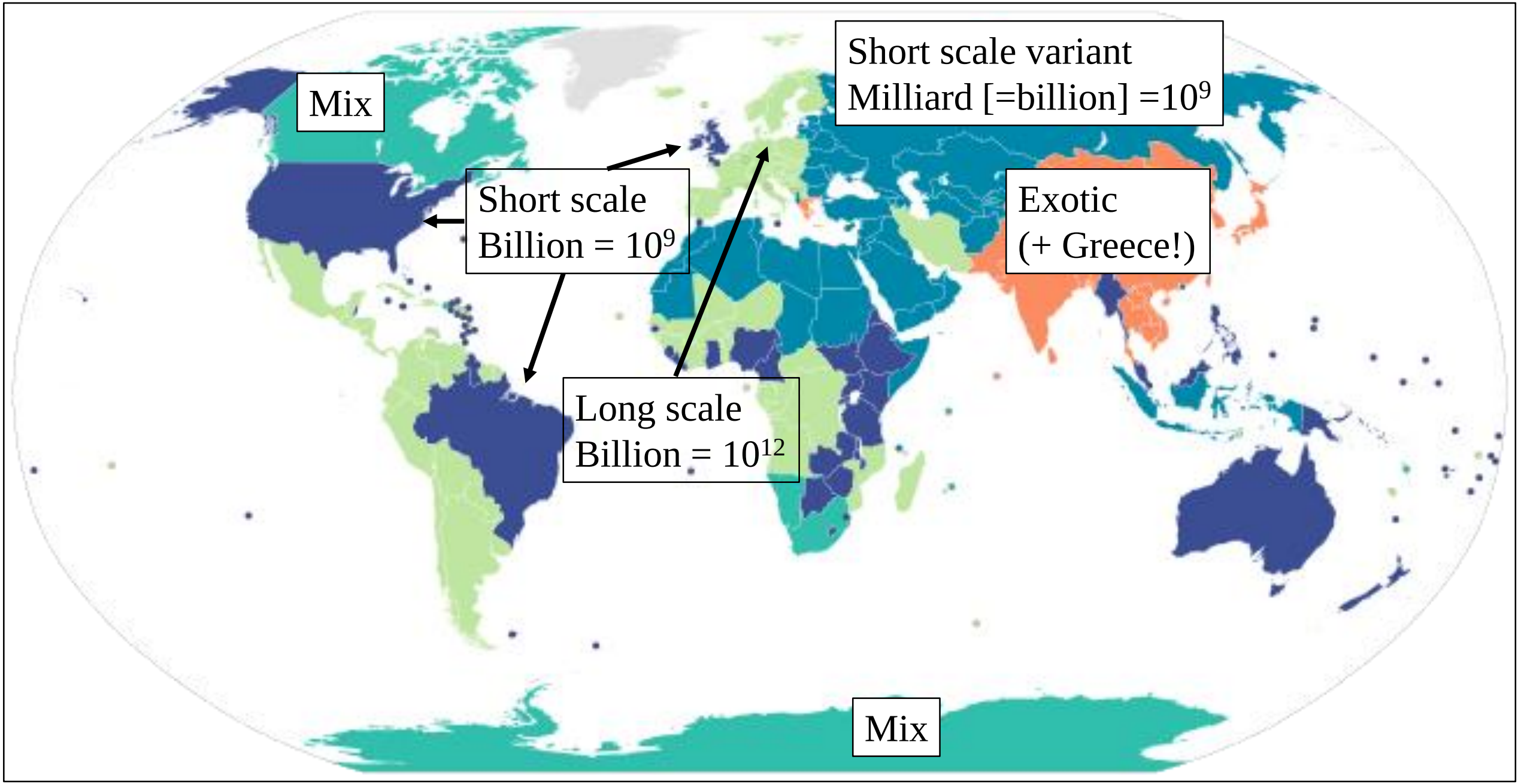
Usage:		
	France 1500 – 1800, ≥ 1961; milliard = 10^9 is also in use. Great Brittan < 1974 West Europe (besides GB)…	France 1800 – 1948; the USA > 1800; Great Brittan ≥ 1974; Bulgaria, Russia, Turkey…

Counting: 1 = one; 10 = ten; 100 = hundred; 1000 = thousand [mille]; 1 000 000 = million

THE LONG SCALE TODAY The step is 1 million = 10^6	**THE SHORT SCALE TODAY The step is 1000**	**SI The step is 1000** Deca = 10^1, Hepta = 10^2, Kilo = 10^3, Mega = 10^6
Billion = 10^{12}	Billion [= milliard] = 10^9	Giga = 10^9
Trillion = 10^{18}	Trillion = 10^{12}	Tera = 10^{12}
Quadrillion = 10^{24}	Quadrillion = 10^{15}	Peta = 10^{15}
Quintillion = 10^{30}	Quintillion = 10^{18}	Exa = 10^{18}
…	…	…

Usage:			
	France 1500 – 1800, ≥ 1961; milliard = 10^9 is also in use. Great Brittan < 1974 West Europe (besides GB)…	France 1800 – 1948; the USA > 1800; Great Brittan ≥ 1974; Bulgaria, Russia, Turkey…	Global besides the USA, Liberia, and Myanmar

The World Map of Exponents



Hungary, 1946:

b.-pengő = billion pengő = 10^{12} pengő
(long scale)

10 000 b.-pengő = 10^{16} pengő
= 10 quadrillion pengő
(short scale)

A modest amount
presented to Vakarelov:

10 000 000 000 000 000!

TRADITIONAL CARDINAL NUMBERS

Ones: one, two,…, nine  10^1

Tens: ten, “2-tens” (twenty, двадесет),…, “9-tens” (ninety, деветдесет)  “10-tens” = 100 =10^2

Hundreds: a hundred, “2 hundreds”,…, “9 hundreds”  “10 hundreds” = 1000 = 10^3

Missing “ten hundreds”,…, “ninety hundreds”;

A new name for “100 hundreds” = 10 000 = 10^4 is needed!

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Thousands: a thousand, “2 thousands”,…, “999 thousands”  “1000-thousands” = million = 10^6

Millions, short scale: a million,…, “999 millions”  “1000-millions” = billion (milliard) = 10^9

Missing: “a thousand million” (*optional*),…, “100 thousand million” = 10^{11}

Millions, long scale: … “999 999 millions”  “million millions” = billion = 10^{12}

TRADITIONAL CARDINAL NUMBERS

Ones: one, two,…, nine STOP 10^1

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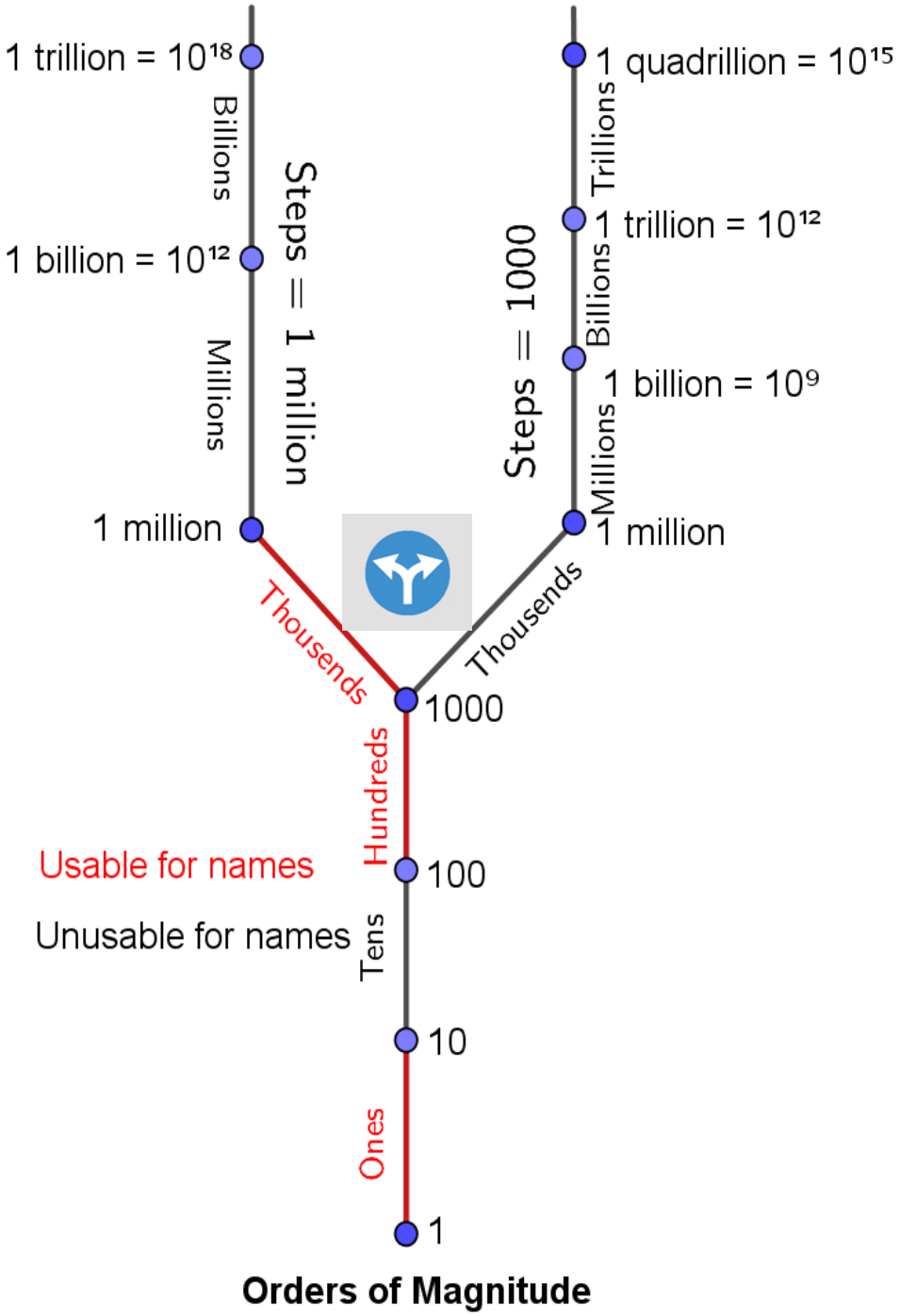
* * *

Short scale (USA, Great Brittan, Bulgaria): 267 billions (milliards) | 351 millions | 759 360

Long scale (France): 450 217 billions || 810 123 millions ||190 220

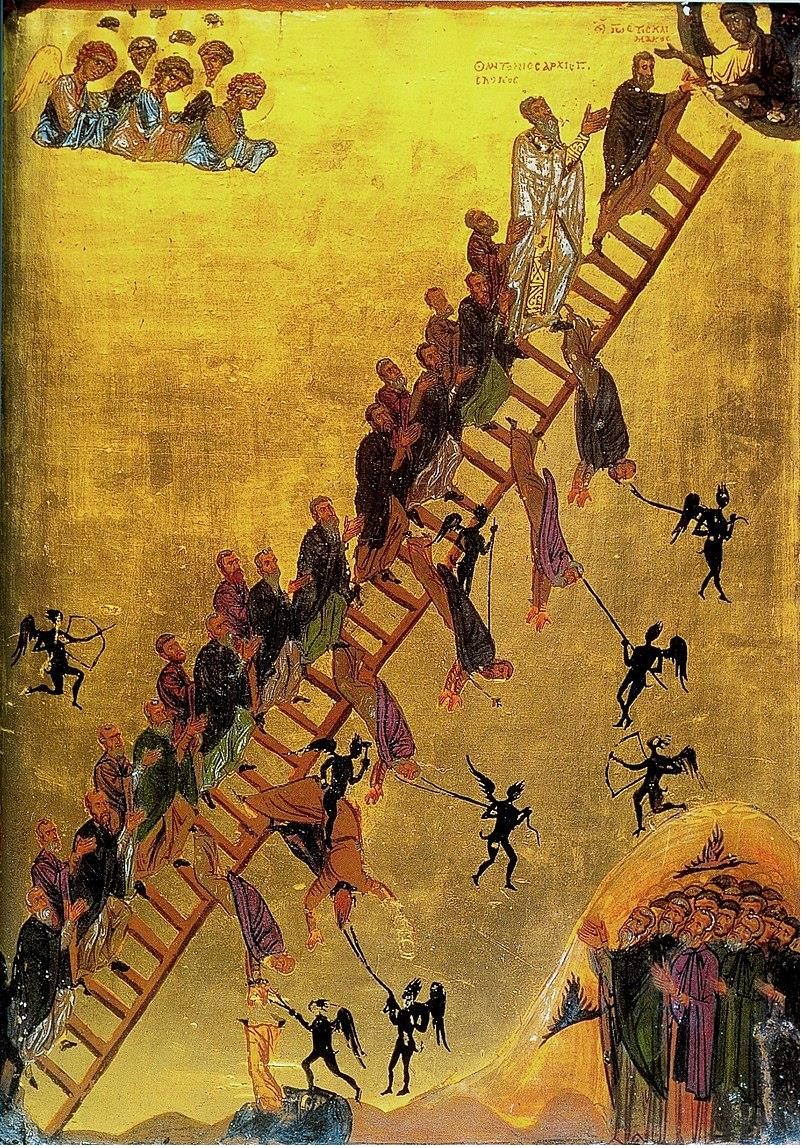
or: 450 billiards | 217 billions || 810 milliards | 123 millions ||190 220



Observations on Numerals

How are they constructed?

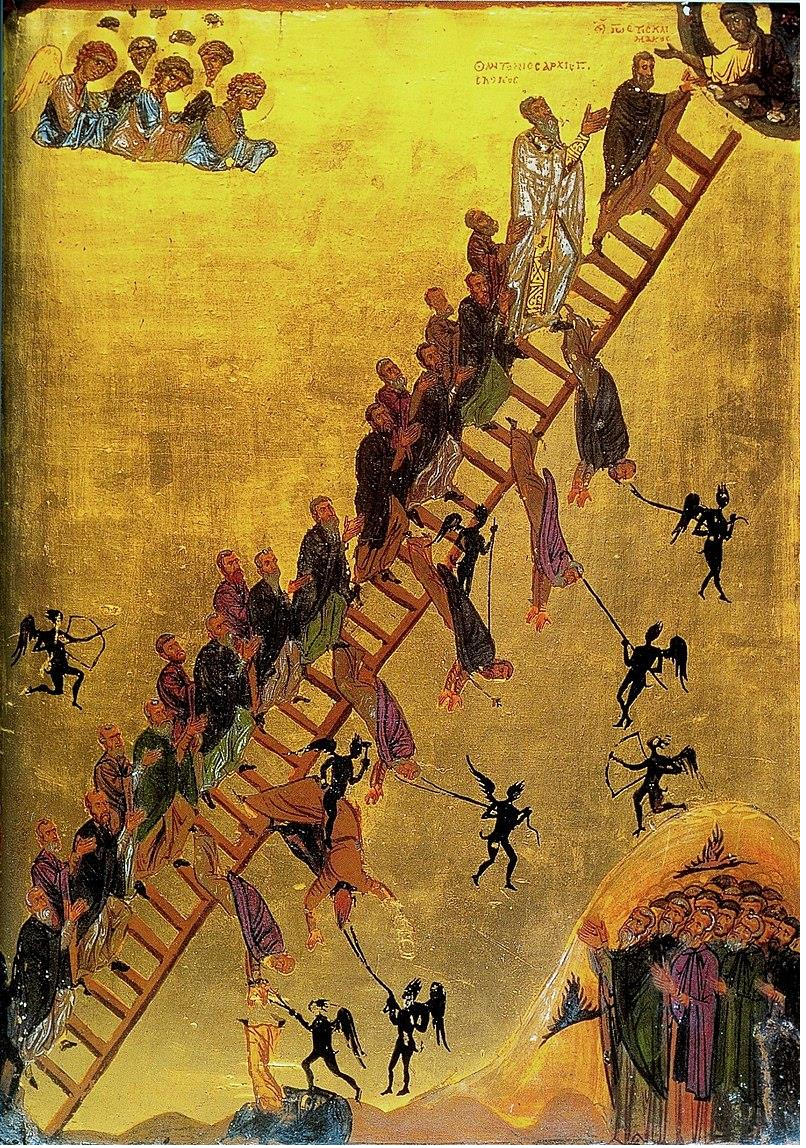
1) The names of the digits are autochthone: *one*, *two*,…
2) The names of the orders ≤ *millions* are autochthone: *tens*, *hundreds*, *thousands*, *millions*, [*milliards*].
3) The common construction of the numerals is a combination of the name of a certain numeral with the name of the next higher order: *thirty* = 3*tens, 200 000 = 200*thousands… Squares (ten*tens) are forbidden.
4) The red orders utilize all their capacity: ones (5*tens), hundreds (200*thousands). The **black** orders utilize their capacity only partially (4*hundreds but not 42*hundreds). The third group depends on the scale: thousands produce 100 000*millions on the long scale but only hundreds*millions on the short one.
5) **The general conclusion:** both scales are linear having constant steps (a thousand on the short scale, and a million on the long one).



The Ladder of Divine Ascent
Sinai, 12th c.
(The linear scales of numerals)

SOTIROV'S SCALE

The steps increase in a geometric progression.



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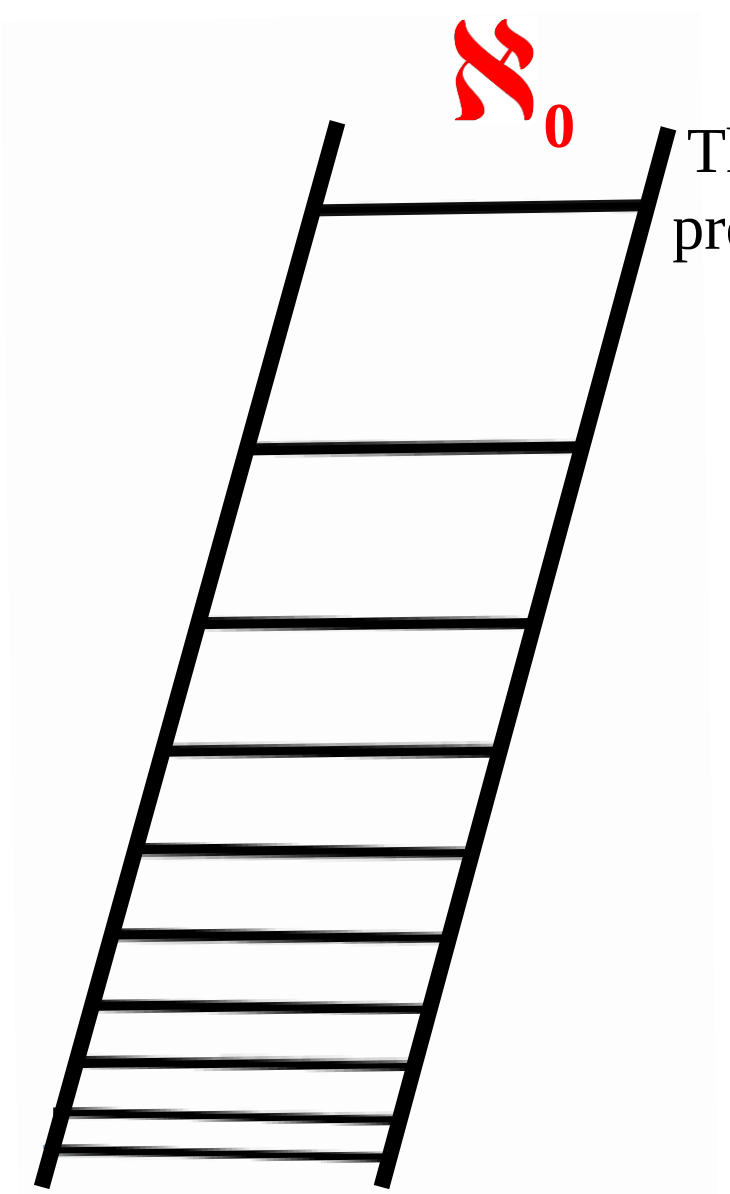

The Innovative Ladder to Infinity

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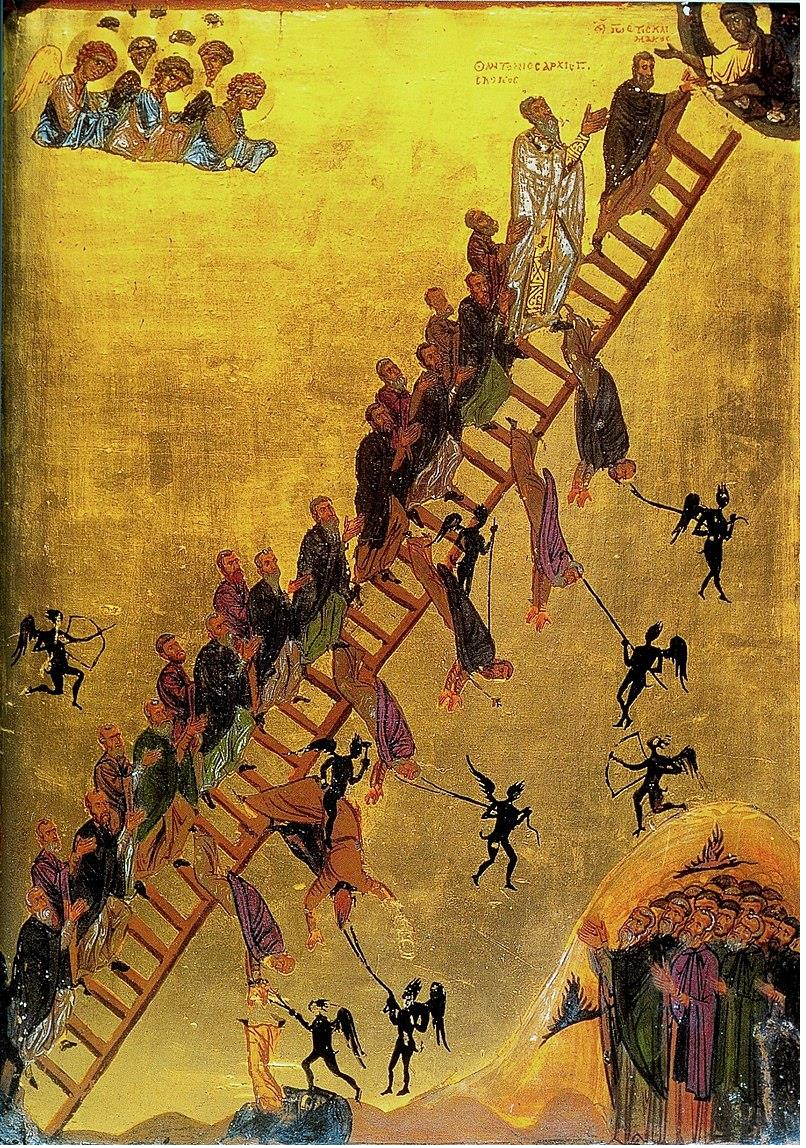
The steps increase in a geometric progression.

The tradition is kept in the law orders:

$1 = N_0$, $10 = N_1$, $100 = N_2$, $1000 = N_3$, $10^6 = N_4$ (*million*), $10^{12} = N_5$ (*billion*).



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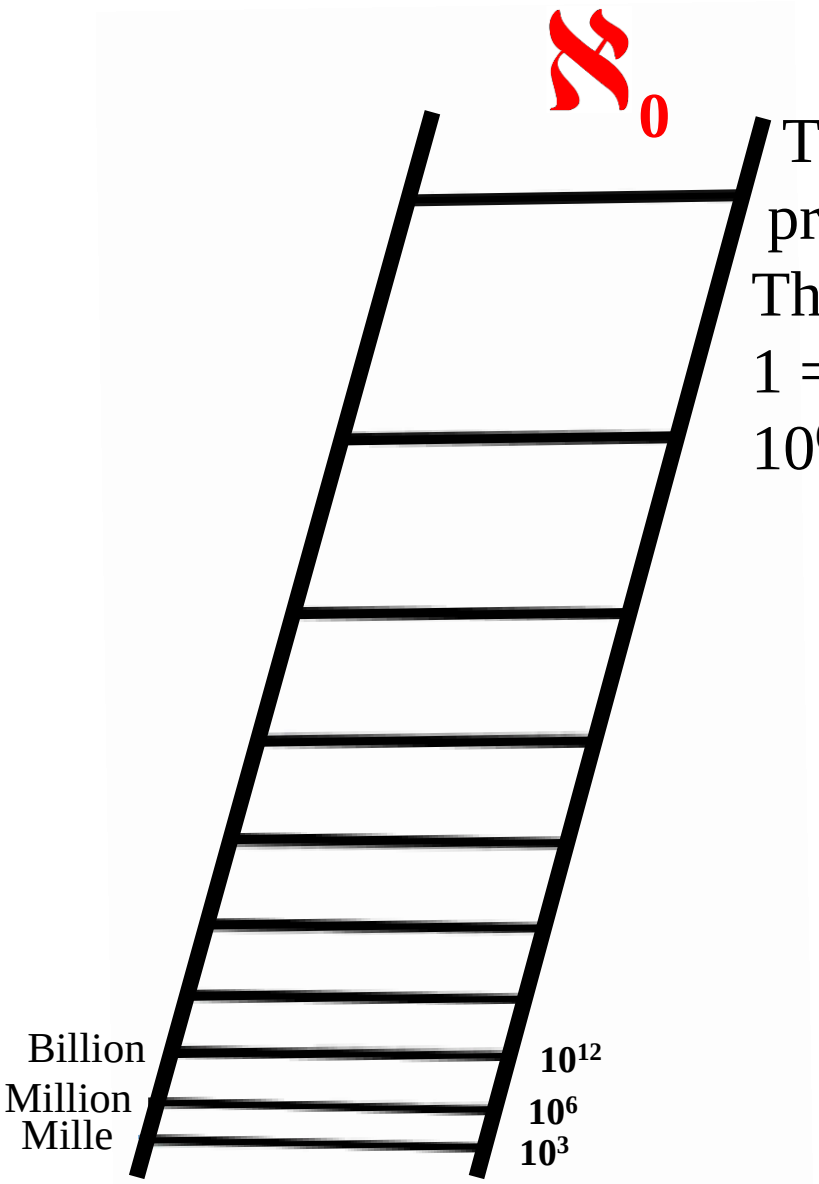

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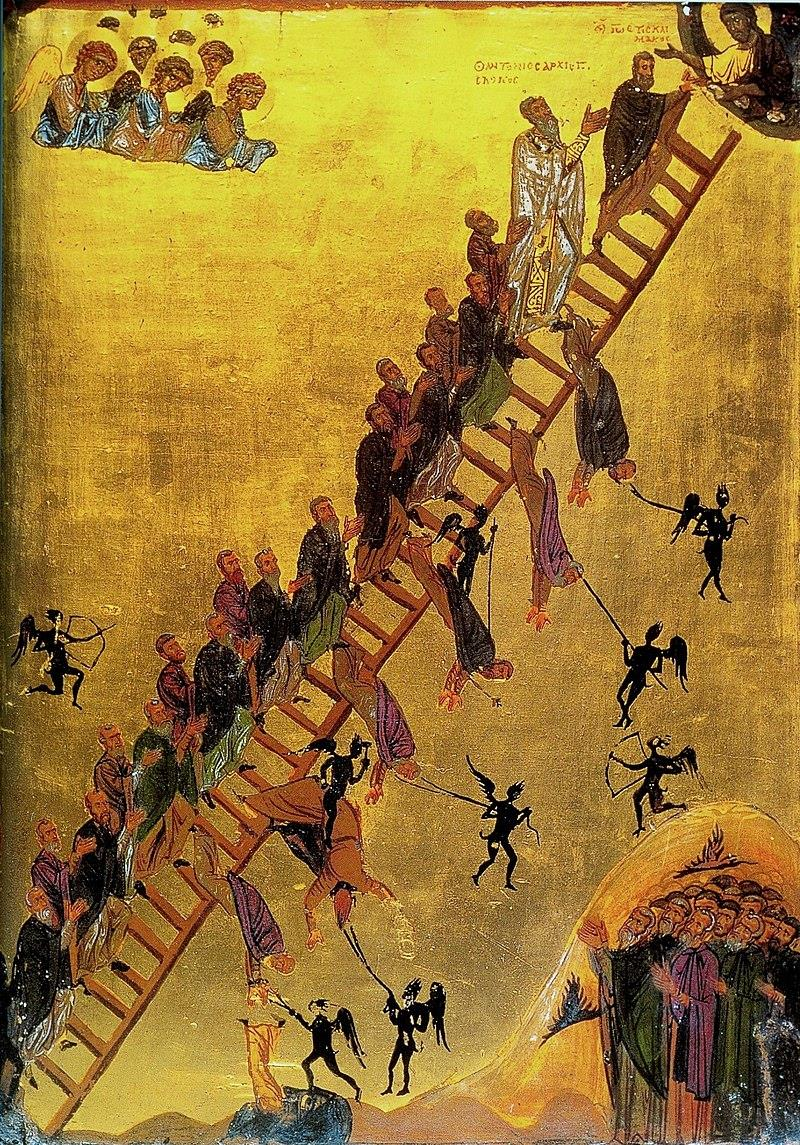

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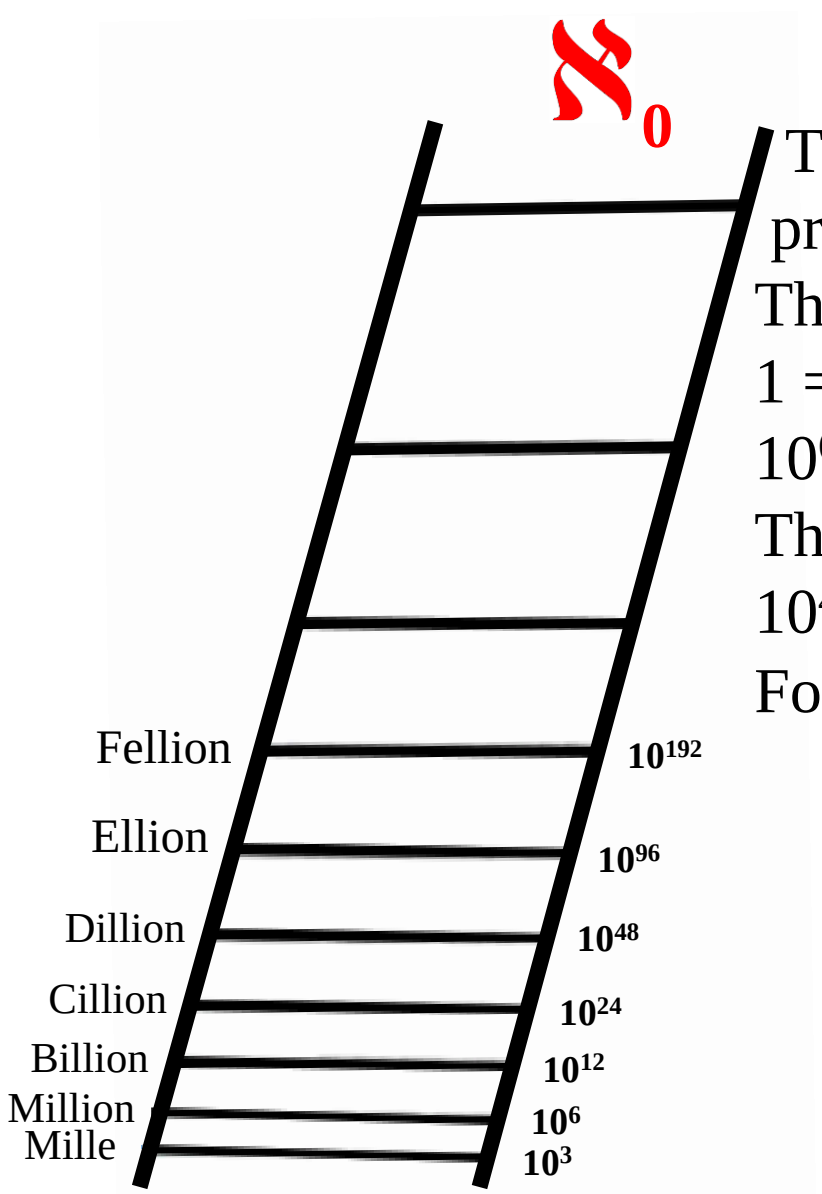

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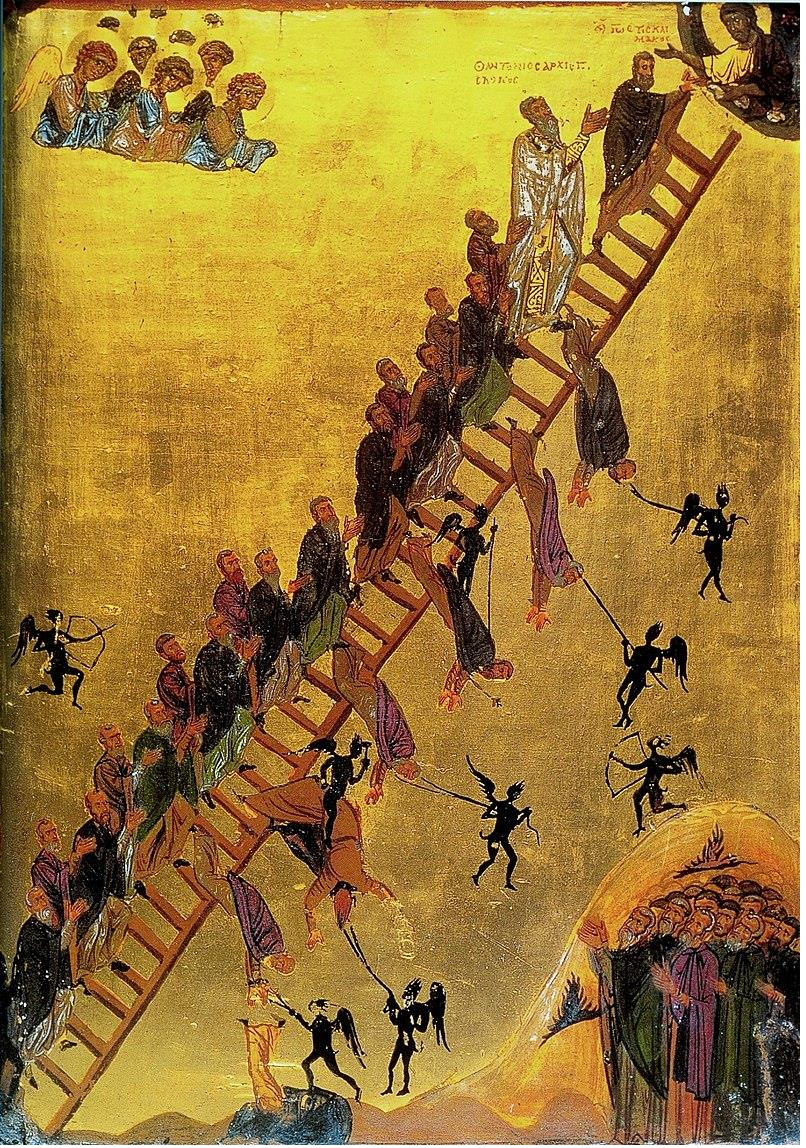
The higher orders: $10^{24} = N_6$ (*cillion*), $10^{48} = N_7$ (*dillion*), etc.

For $k \geq 3$, $N_{k+1} = N_k^2$.



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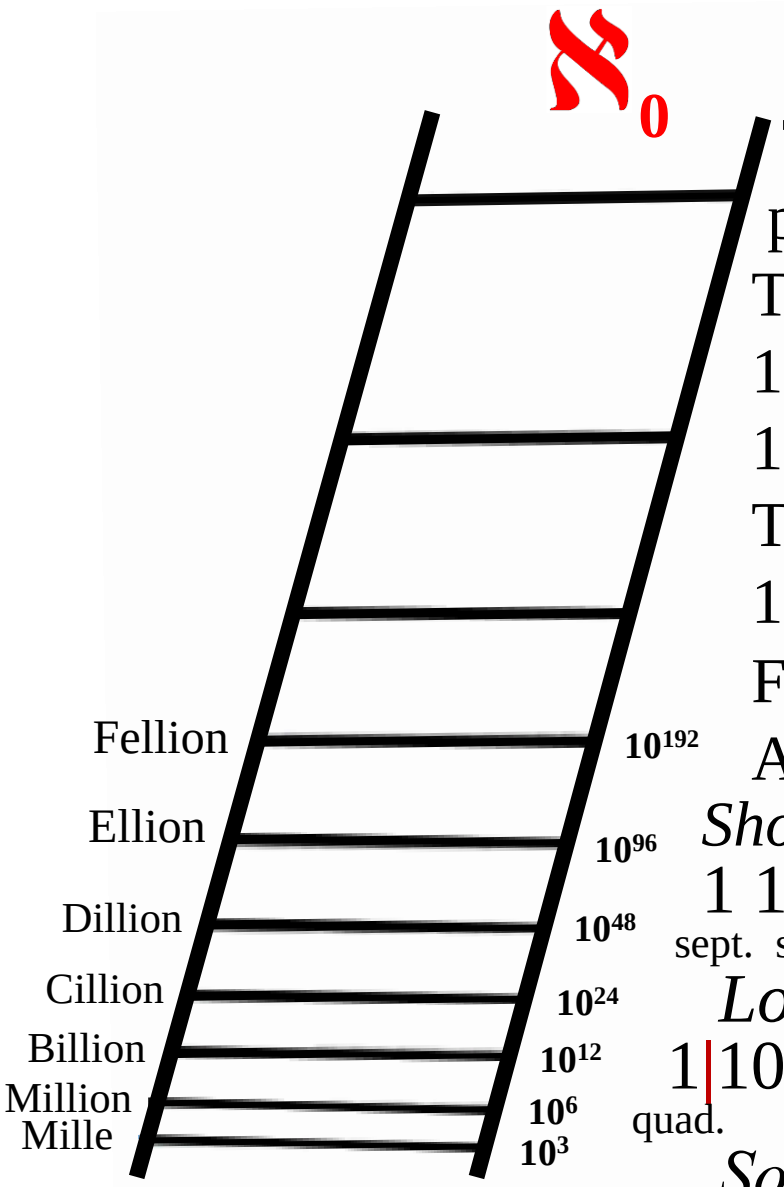

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A comparative example:

Short scale

1 100 100 100 100 100 100 100 100
sept. sixt. quint. quad. trill. bill. mill. thous. hund.

Long scale:

1|100 100|100 100|100 100|100 100
quad. trillion long billion million thous. hund.

Sotirov's scale:

1|100 100 100 100|100 100|100 100
cillion long billion million thous. hund.

The result: 4 (1) numerals saved!

TWO REMARKABLE THEOREMS

Definitions: $N_1 = 10$; $N_2 = 100$; $N_3 = 1000$; $N_{k+1} = N_k^2$ for $k \geq 3$.

Theorem 1. For $k \geq 3$, $N_k = 10^{3.2^{k-3}}$.

Proof (*by induction*): For $k = 3$, $N_3 = 10^3$.

Let $N_k = 10^{3.2^{k-3}}$. Then $N_{k+1} = \left(10^{3.2^{k-3}}\right)^2 = 10^{3.2.2^{k-3}} = 10^{3.2^{(k+1)-3}} = N_{k+1}$, **Q.E.D.**

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Theorem 2. For $k \geq 2$, $\prod_{i=1}^{k} N_i = N_{k+1}$.

Proof using sums of the geometric progression of the exponents: Exercise!

Proof (*evident by induction*): $N_1.N_2 = 1000 = N_3$;
$\prod_{i=1}^{k+1} N_i = N_{k+1}.\prod_{i=1}^{k} N_i = N_{k+1}.\ N_{k+1} = N_{k+2}$, **Q.E.D.**

The Long Life of the Myriad (-? – …)

It is the missing link in the actual scales.
In Ancient Greece: 1 *myriad* = 10 000 = 10^8;
a *myriad myriads* = **∞.**

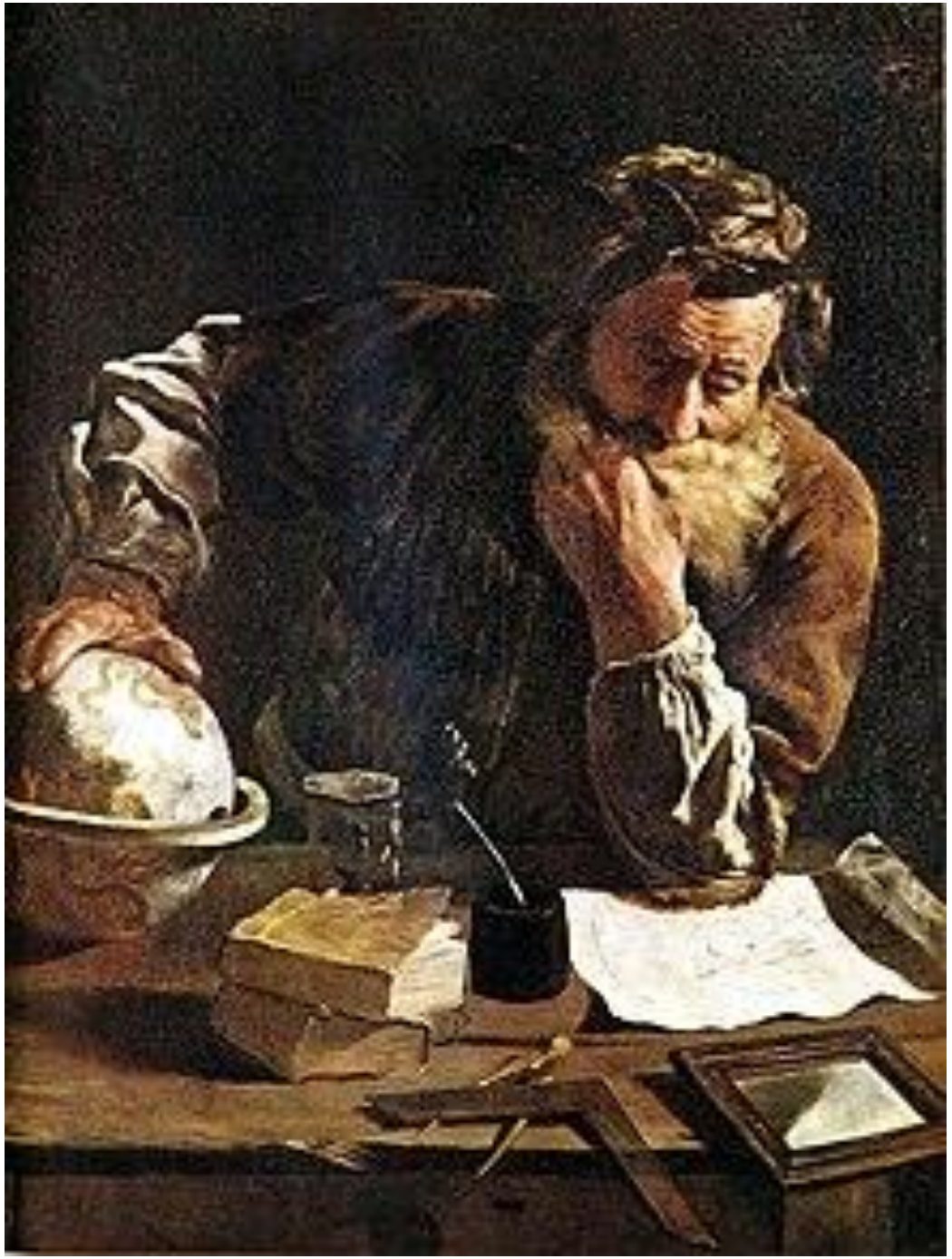
An unexpected relict: a border stone
between Austria and Italy, 1910.
1 Myriameter = 1 kilometer.

The Beginning of the Tale: Archimedes (257 – 212)

The Sand Reckoner (Ψαμμίτης): Constructing and naming unimaginable big numbers, up to

$$\left((10^8)^{(10^8)}\right)^{(10^8)} = 10^{8 \cdot 10^{16}}.$$

His scale follows a geometric progression!



The Chinese hieroglyph for 10 000:

Its celebrating meaning (Japan): *Banzai*!
(10 000 *years of life*!).

The Bulgarian analog:
Да илядиш!
(Only 1000 years, but enough.)

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Vakarelov's catch is a happy omen: 10 000 years to him!

10*x*!