Subrecursive Computability in Analysis
Master Thesis
by Ivan Dimitrov Georgiev

The major goal of the thesis is to indicate that many constants in mathematics can be computed approximately by means of very simple algorithms. The exact definition is given in an article by prof. Skordev in year 2002 and concerns computability of real numbers with respect to a class $\mathcal{F}$ of total functions in the natural numbers. The role of $\mathcal{F}$ in the thesis is taken by the three subrecursive classes $\mathcal{M}^2$, $\mathcal{L}^2$ and $\mathcal{E}^2$.

The thesis is separated into five chapters. The first chapter is introductory. The second chapter includes many propositions, which are auxuliary and prepare the grounds for the forthcoming results. In the same chapter an algorithm for proving $\mathcal{L}^2$-computability and $\mathcal{E}^2$-computability is formulated, which is proven by prof. Skordev in an article in year 2008. The adjustment of this algorithm for $\mathcal{M}^2$-computability is designed and proven by the author of the thesis.

The third chapter is devoted to continued fractions. The main result here is that if we have a sequence of positive integers, which belongs to $\mathcal{E}^2$, then the infinite continued fraction, generated by it, is a real number, which is $\mathcal{E}^2$-computable.

The fourth chapter contains proofs for $\mathcal{L}^2$-computability of many famous constants such as the constant of Euler-Mascheroni, the constant of Mertens, Lebesgue constants and more. It also contains a proof that the Riemann's zeta function, restricted to natural numbers, is $\mathcal{L}^2$-computable.

The last chapter concerns $\mathcal{M}^2$-computability of some famous constants. It begins with $\mathcal{M}^2$-computability of the number e and the Liouville's number L (facts, which are also proven by prof. Weiermann, but in another way). Several proofs follow, which ground that the constant of Erdös-Borwein, the number π, the logarithm of the golden mean φ and the paper folding constant σ are $\mathcal{M}^2$-computable.